Calculus IV
(MultiVariable or Vector Calculus)
Detailed Syllabus

Mathematics

- Vectors: How they are plotted, how they are moved, how they are added and how they are multiplied by numbers.
- Tangent vectors, velocity vectors and tangent lines.
- Dot product of two vectors and the component (push) of one vector in the direction of another.
- Cross product and dot products.
- Planes in three dimensions.
- Normal vectors for surfaces in three dimensions.

Science and Math Experience

- Bouncing light beams off curves in two dimensions.
- Experiments dealing with reflecting properties of parabolas and ellipses.
- Pursuit models.
- Parametric formulas for a line.
- Laser zapping along the tangent line.
- Velocity and acceleration vectors.
- Analyzing an object's motion and speed by looking at tangential components of its acceleration.
- Normal and tangential components of acceleration.
- Plotting the motion of the Earth-Mars-Sun system with Earth at the origin.
- Stealth technology.
- Flatness and plotting.
- Using the main unit normal and binomial as moving frames to plot tubes, horns, ribbons and corrugations centered on a given curve.
- Experiments with linearizations (tangent plane approximations).
- Bouncing light beams off surfaces in three dimensions.
- Kissing circles and curvature in two and three dimensions.
- Boring holes with a robotic router.

Mathematics

- The gradient and the chain rule. Level curves, level surfaces and the gradient as normal vector.
- The gradient as a vector pointing in the direction of greatest initial increase.
- Linearizations and the chain rule.
- Total differential.
- Lagrange multipliers.
- Vector fields as fluid flow.
- Trajectories in vector fields as the path a cork floats on.
- Solutions of differential equations y'[x] = f[x,y[x]] as trajectories in a vector field.
- Path integrals (line integrals) as measurements of the net flow of a given vector field across a given curve. Path integrals (line integrals) as measurements of the net flow of a given vector field along a given curve.
- Path independence and gradient fields.
- Recognition of gradient fields.

Science and Math Experience

- Using the gradient for maximization and minimization.
- Ascent and descent paths through the gradient field with application to the idea behind finding minimum instructions.
- Vibrating string and one dimensional heat.
- Programming heat seeking missiles.
- Cobb-Douglas manufacturing model.
- Experience with Beale's valley function, Rosenbrock's banana function.
- Optimization problems from metallurgy.
- Duffin's barge problem.
- Tangential and normal components of vector fields on given curves.
- Visual experiments dealing with the net flow of vector fields across given curves.
- Visual experiments dealing with the net flow of vector fields along given curves.
- The 2D electric field. Dipoles in 2D.
- The gradient field.
- Experiments with how gradient fields look near maximizers and minimizers.
- Where trajectories in the gradient field want to go.
- Where trajectories in the negative gradient field want to go.
- Looking for spigots and drains by following the path of a cork.
- Logistic harvesting model.
- Sources and sinks of 2D vector fields.
- Sources and sinks at singularities in vector fields.
- Work as flow in a force field.
- Which way to go to make the a force field do most of the work.
- Force fields and their trajectories.
- Models for water flow.
- Clockwise versus counterclockwise flow.

Mathematics

- Divergence and rotation of a 2D vector field.
- Sources as points at which the divergence is positive; sinks as points at which the divergence is negative.
- Using the Gauss-Green formula to measure the flow of a 2D vector field across a closed curve by means of a 2D integral.
- Using the Gauss-Green formula to measure the flow of a 2D vector field along a closed curve by means of a 2D integral.
- Singularity sources, sinks and swirls. uv paper and xy paper when u = u[x,y] and v = v[x,y].
- The uv grid on xy paper.
- Linearizing the uv grid on xy paper.
- The Jaocbian as the area conversion factor for converting xy paper area measurements into uv paper area measurements.
- Transforming 2D integrals: How it is done and why it is done.
- The 3D integral f[x,y,z] dx dy dz via slicing and accumulating. dx dy dz as a volume measurement.
- Average value of a function on a region.
- The Jacobian as a local volume conversion factor in 3D.
- Transforming 3D integrals.
- Mass and density.

Science and Math Experience

- Why it is that if all points inside a given closed curve C are sources of a given 2D vector field, then the net flow of the vector field across C must be from inside to outside.
- Encapsulating singularities with small circles centered on the singularity.
- Flow measurements in the presence or absence of singularities. 2D electric fields.
- Dipole fields.

- Gauss's law for calculating the flux of combined 2D electric fields.
- Parallel flow.
- The Laplacian as the divergence of the gradient field.
- Why harmonic functions cannot have local maxima or minima.
- Steady state heat and the Laplace's equation in two dimensions.
- Spin fields.
- Semi-log and log-log paper.
- Flow measurements for 2D vector fields.
- If the boundary of a region can be plotted with Mathematica, then the plotting instructions usually carry enough information to make it possible to measure the area of the region.
- Why crazy things are likely to happen when the area conversion factor is 0.
- Analyzing a transformation by plotting its area conversion factor and by plotting the gradient field of its area conversion factor.
- Experiments relating linear equations and area measurements in two dimensions.
- Experiments with eigenvectors as stretching directions and eigenvalues as stretching factors in linear transformations in two dimensions.
- If the whole skin of a solid region can be plotted with Mathematica, then the plotting instructions usually carry enough information to make it possible to measure the volume of the solid region.
- Cylinders, spheres and tubes:
- Plotting them and integrating on them.
- Integrating on solids bounded by sets of surfaces.
- Switching the order of integration.
- Tubes, horns and squashed doughnuts.
- Drilling and slicing spheres.
- Experiments relating linear equations to volume measurements.
- Centroids and centers of mass.
- Bidding on rocket nose cones.

Mathematics

- Meaning of each of the spherical parameters.
- Plotting and measuring volumes of spheres ellipsoids, cones and measuring volumes of each.
- Integration with spherical coordinates.
- Measuring area on surfaces.
- Surface integrals for measuring flow of 3D vector fields across surfaces.
- Sources, sinks and Gauss's formula in 3D.
- Measuring flow along a 3D curve with a path integral.
- The curl of a 3D vector field.
- Orientation and Stokes's formula.
- Stokes's formula as an outgrowth of the 2D Gauss-Green formula.

Science and Math Experience

- Earth-Moon animation's. Snail shells. Star Wars window of vulnerability plots adapted from NASA work. Using spherical coordinates to design and paint flowers.
- Centering and aligning the general 3D ellipsoid.
- Ice cream cone and tops.
- Passing a plane between two disjoint solid disks.
- Measuring the volume inside 4D spheres.
- Experiments with eigenvectors as stretching directions and eigenvalues as stretching factors in linear transformations in three dimensions.
- Measuring flow across surfaces: Gauss's formula versus calculation by a surface integral.
- Substitute surfaces to avoid calculational nightmares.
- Encapsulating singularities with small spheres centered on the singularity.
- 3D electric fields and Gauss's law for calculating the flux of combined 3D electric fields.
- The Laplacian as the divergence of the 3D gradient field.

- Sources and sinks in the 3D gradient field and their relation to max-min.
- Why harmonic functions of three variables cannot have local maxima or minima.
- Steady state heat in a solid.
- Morphing and Moebius strips.
- Fingering a 3D vector field.
- The curl as the axis of the greatest counterclockwise swirl.
- Paddle wheels.
- Parallel flow and irrotational flow.
- Ideal fluid flow.
- Work done by 3D force fields.
- Recognition of 3D gradient fields.
- Path independence.

*Note: This description covers both Math 262 and Math 265. Math 262 will cover about 75% of the material that math 265 will cover (3 credit course versus 4 credit course).

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