Matrices, Geometry&*Mathematica*

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MGM.01 Perpendicular Frames
GIVE THEM A TRY!

Mathematica Initializations

Experience with the starred problems will be very useful when you go into later lessons.

RC: 10/20/07: A little to finish up here.

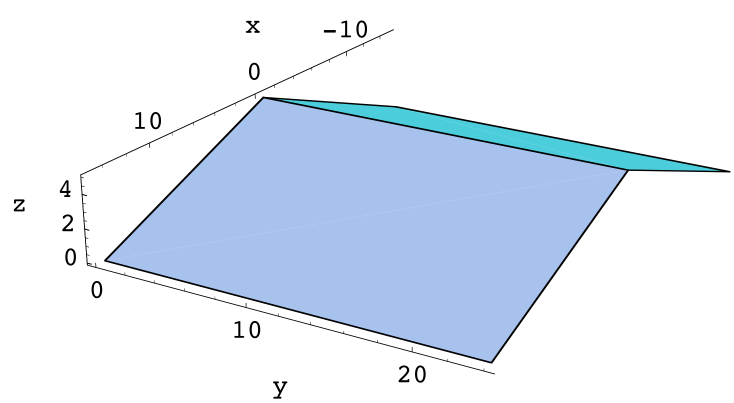
RC: 10/25/07: Looking good. Notebook complete.

G.9) Stovepipes and roofs

G.9.a) Stovepipes and roofs

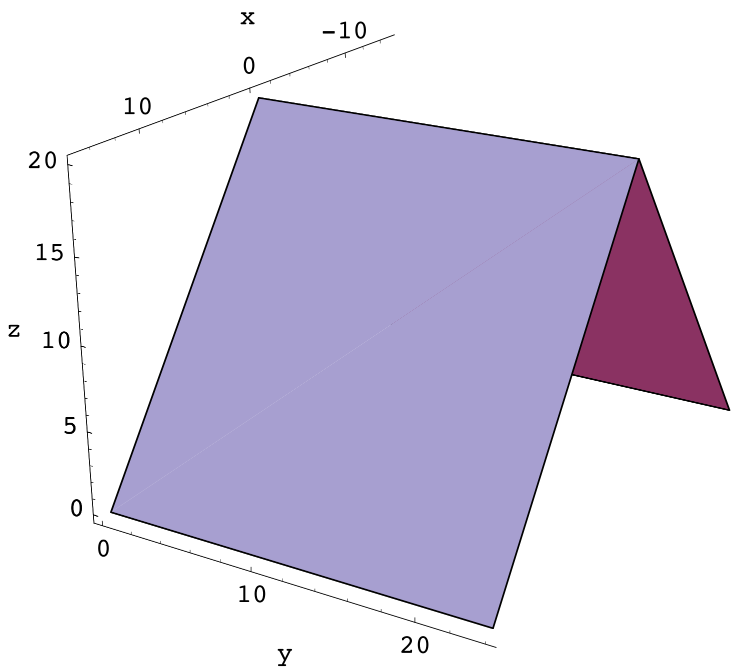
Here is a roof top:

```
Clear[roof, x, y, z];
    roof[x_, y_, z_] :=
  {Graphics3D[Polygon[{{x, 0, 0}, {x, y, 0}, {0, y, z}, {0, 0, z}, {x, 0, 0}}]],
        Graphics3D[Polygon[{{-x, 0, 0}, {-x, y, 0}, {0, y, z}, {0, 0, z}, {-x, 0, 0}}]]};
Show[roof[15, 24, 5], ViewPoint -> CMView,
  Axes -> True, Boxed -> False, AxesLabel -> {"x", "y", "z"}];
```

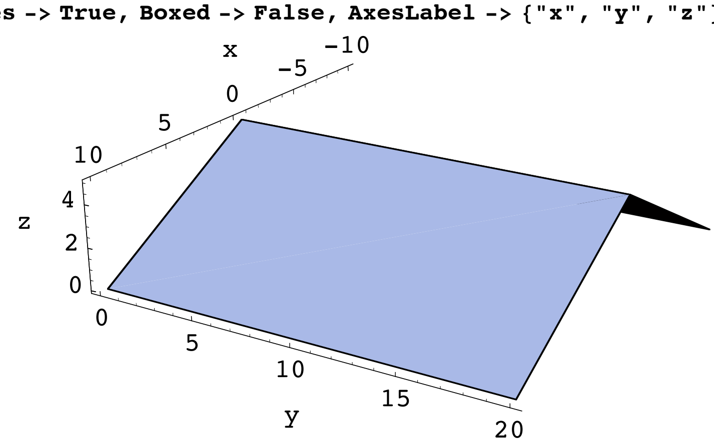


The meaning of the variables x,y and z are given in the plot.
See some more:

```
Show[roof[15, 24, 20], ViewPoint -> CMView,
  Axes -> True, Boxed -> False, AxesLabel -> {"x", "y", "z"}];
```



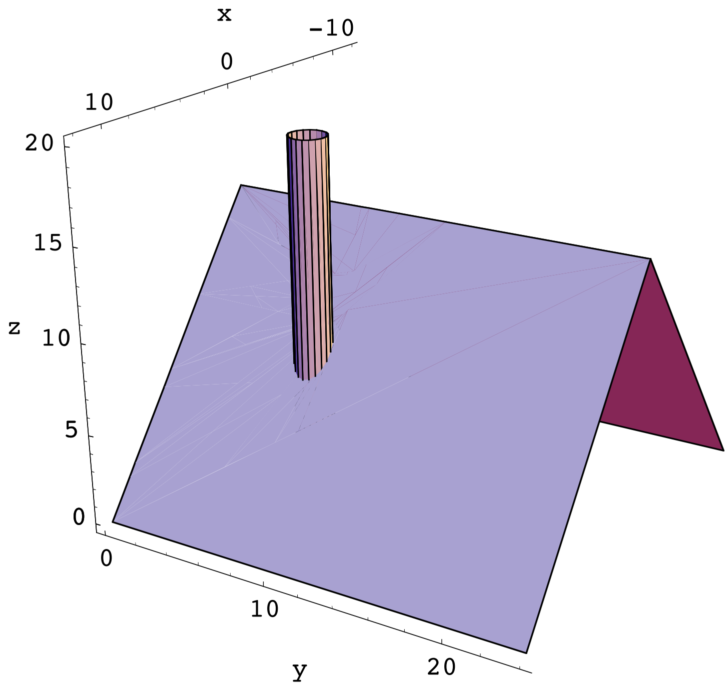

```
Show[roof[10, 20, 5], ViewPoint -> CMView,
  Axes -> True, Boxed -> False, AxesLabel -> {"x", "y", "z"}];
```



Here comes a stove pipe through one of these roofs:

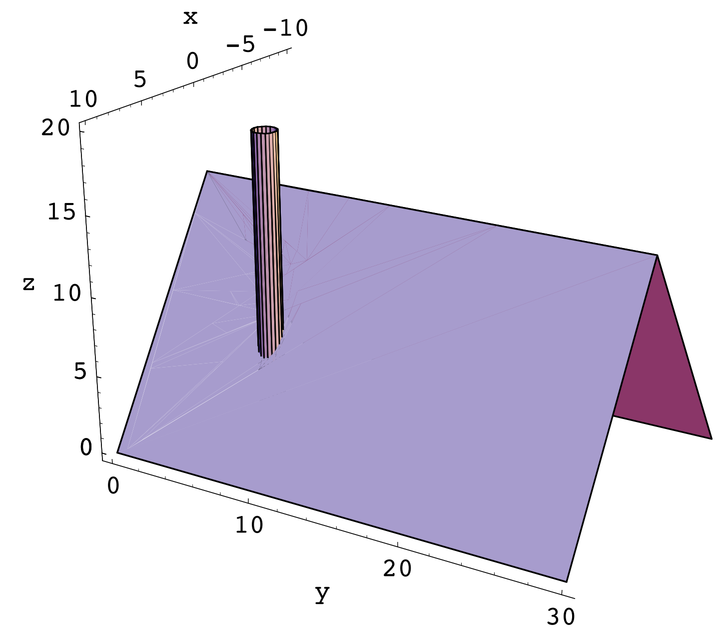
```
Clear[stovepipe, x, y, z, radius];

stovepipe[x_, y_, z_, radius_] :=
  ParametricPlot3D[s ({x, y, 0} + radius {Cos[t], Sin[t], 0}) +
    (1 - s) ({x, y, z}  + radius {Cos[t], Sin[t], 0}) ,
        {s, 0, 1}, {t, 0, 2 Pi}, PlotPoints -> {2, Automatic},
   DisplayFunction -> Identity];
x = 12;
```

```
y = 24;
z = 15;
xstovepipecenter = 5;
ystovepipecenter = 8;
stovepipeheight = 20;
stovepiperadius = 1;

roofplot = Show[roof[x, y, z], stovepipe[xstovepipecenter,
    ystovepipecenter, stovepipeheight , stovepiperadius], PlotRange -> All,
   ViewPoint -> CMView, Axes -> True, Boxed -> False, AxesLabel -> {"x", "y", "z"}];
```



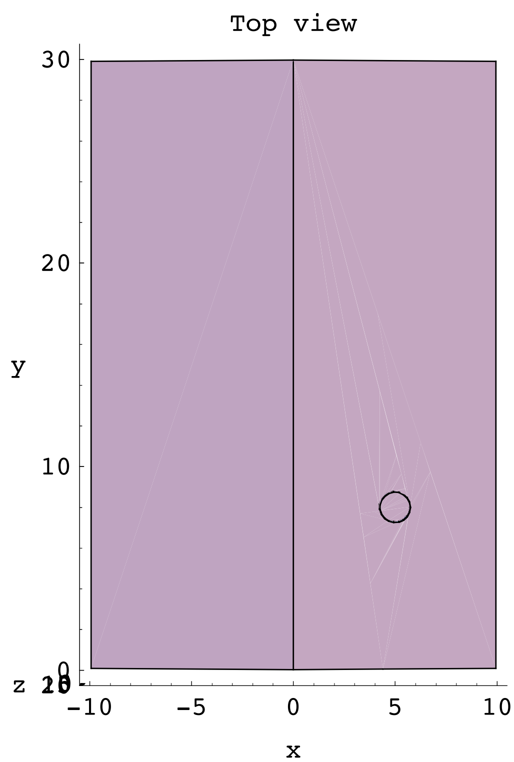
Another:

```
x = 10;
y = 30;
z = 15;
xstovepipecenter = 5;
ystovepipecenter = 8;
stovepipeheight = 20;
stovepiperadius = 0.75;

roofplot = Show[roof[x, y, z], stovepipe[xstovepipecenter,
    ystovepipecenter, stovepipeheight , stovepiperadius], PlotRange -> All,
   ViewPoint -> CMView, Axes -> True, Boxed -> False, AxesLabel -> {"x", "y", "z"}];
```

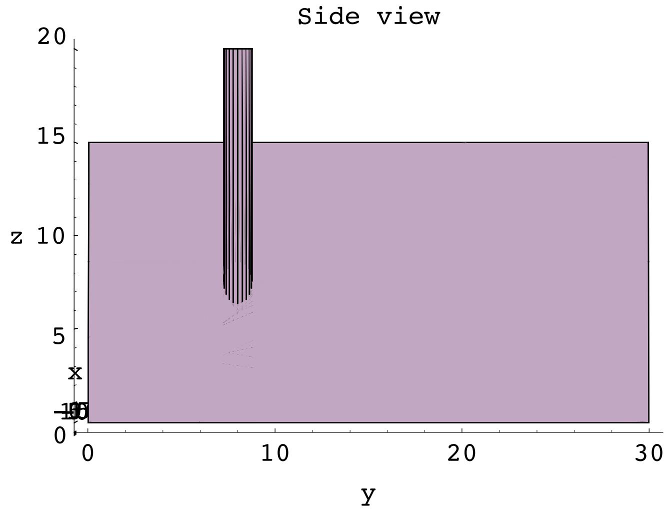


Se the top view, the side view and the end view:

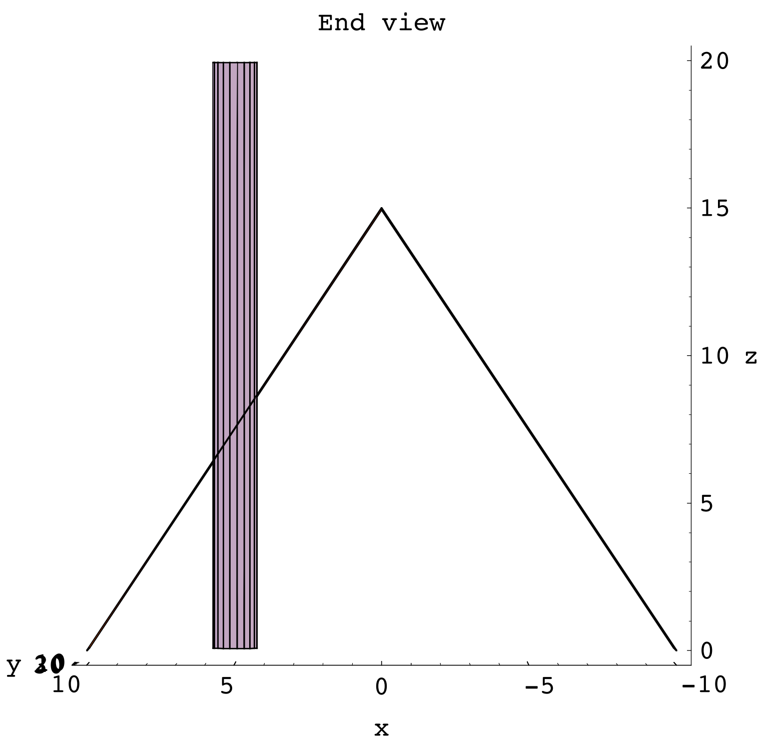
```
topview = Show[roofplot, ViewPoint -> 100 {0, 0, 1}, PlotLabel -> "Top view"];
```




```
sideview = Show[roofplot, ViewPoint -> 100 {1, 0, 0}, PlotLabel -> "Side view"];
```

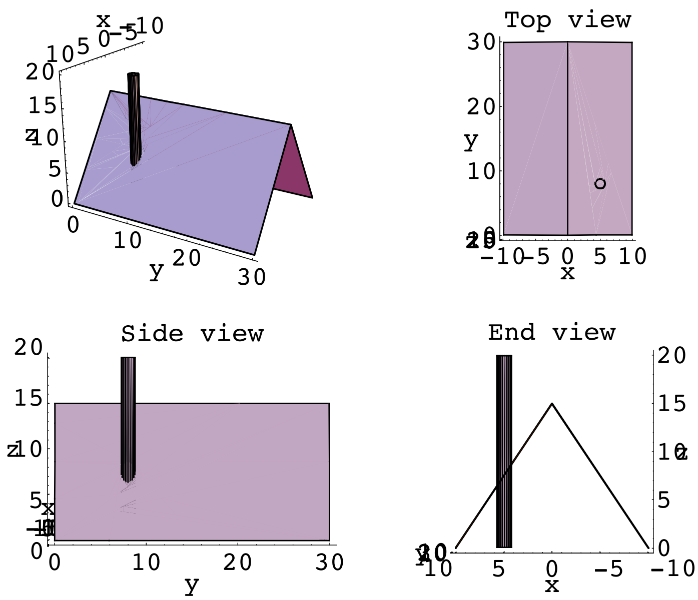



```
endview = Show[roofplot, ViewPoint -> 100 {0, 1, 0}, PlotLabel -> "End view"];
```



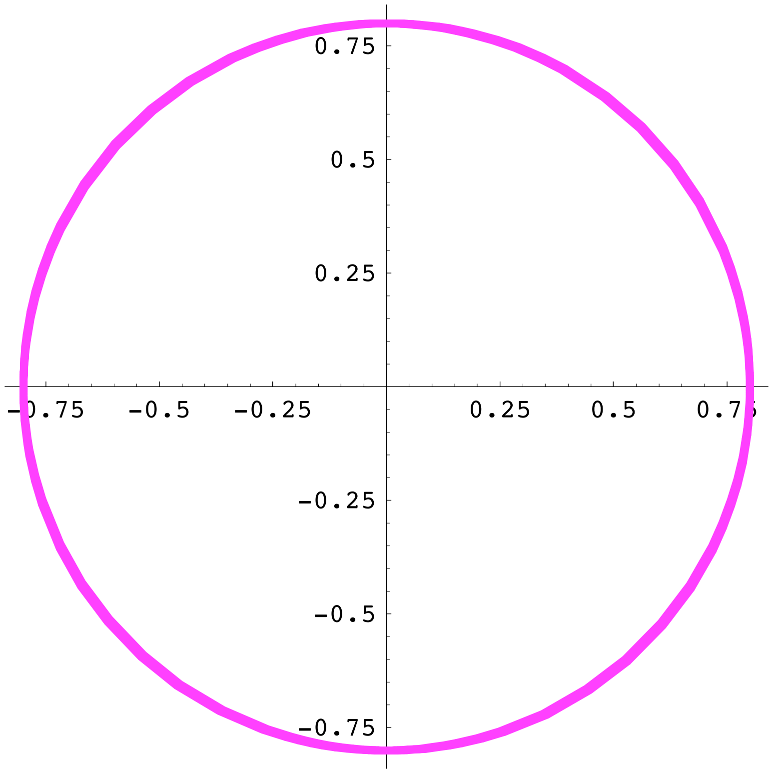
See everything:

```
Show[GraphicsArray[{{roofplot, topview}, {sideview, endview}}]];
```



Your job is to cut a hole in the metal roof to let the stovepipe come through.
That lab pest the dorky Calculus Cal says to cut a circle whose radius is the same as the radius of the stovepipe.
For instance in the plot above, the radius is 0.8; so Cal's suggestion is to cut this circle out of the metal roof sheeting:

```
Calholeplot = ParametricPlot[{0.8 Cos[t], 0.8 Sin[t]},
   {t, 0, 2 Pi}, PlotStyle -> {{Thickness[0.01], Magenta}}];
```


0.75
0.5
0.25
-0.75
-0.5
-0.25
0.25
0.5
0.75
-0.25
-0.5
-0.75

What do you say to Cal?
What curve do you cut out of the metal roof sheeting to let this pipe pass through snugly?

RC: 10/20/07: Careful: the angle of the roof does not increase. The angle of the roof is constant. This roof is
not a ski jump..

The hole in the roof sheeting will have to be an ellipse, because the pipe intersects the plane of the roof at an angle, not perpendicularly. The horizontal axis of the ellipse will be the same as the diameter of the stovepipe, but the vertical axis of the ellipse will be stretched by a factor related to the steepness of the roof: as the angle of the roof increases (becomes more steep), the long axis will lengthen; if you decreased the pitch of the roof toward an angle of 0 (horizontal), the long axis would shorten back to the diameter of the circle.
Here's how I would calculate the length of the long axis ("ystretch"):

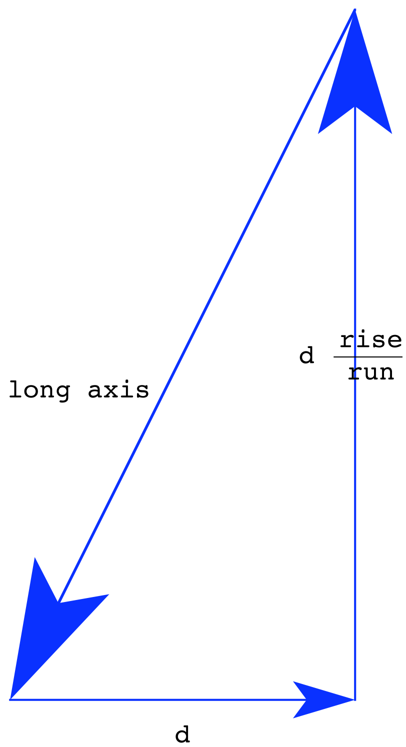
slope of roof = $\frac{\text{rise}}{\text{run}}$ (for example, rise 1.5 m per 1 m horizontal distance)

diameter of pipe = d (for example, 0.1 m)

vertical distance = $d \frac{\text{rise}}{\text{run}}$

ystretch = $\sqrt{d^2 + (d \frac{\text{rise}}{\text{run}})^2}$

```
Clear[point1, point2, point3];
point1 = {0, 0};
point2 = {1, 0};
point3 = {1, 2};
Show[Arrow[point2 - point1, Tail → point1], Graphics[Text["d", {0.5, -0.1}]],
  Arrow[point3 - point2, Tail → point2], Graphics[Text["d rise/run", {1, 1}]],
  Arrow[point1 - point3, Tail → point3], Graphics[Text["long axis", {0.2, 0.9}]]];
```



So using my example dimensions, the long axis of the ellipse would be

RC: 10/20/07: Where does 0.1 come from? I think we have this measurement here correctly given above, incase 0.1 was just "any number".

```
Clear[d, slope];
d = 0.1;
slope = 1.5;
longaxis = Sqrt[d^2 + (d slope)^2]
```

```
0.180278
```

0.18 m (18 cm).

MCB 10/21/07: I was just thinking hypothetically, but using the numbers given for a roof above, the answer would be:

```
Clear[roof, x, y, z];
    roof[x_, y_, z_] :=
  {Graphics3D[Polygon[{{x, 0, 0}, {x, y, 0}, {0, y, z}, {0, 0, z}, {x, 0, 0}}]],
        Graphics3D[Polygon[{{-x, 0, 0}, {-x, y, 0}, {0, y, z}, {0, 0, z}, {-x, 0, 0}}]]};
```

```
Clear[stovepipe, x, y, z, radius];

stovepipe[x_, y_, z_, radius_] :=
  ParametricPlot3D[s ({x, y, 0} + radius {Cos[t], Sin[t], 0}) +
    (1 - s) ({x, y, z} + radius {Cos[t], Sin[t], 0}) ,
        {s, 0, 1}, {t, 0, 2 Pi}, PlotPoints -> {2, Automatic},
   DisplayFunction -> Identity];
x = 12;
y = 24;
z = 15;
xstovepipecenter = 5;
ystovepipecenter = 8;
stovepipeheight = 20;
stovepiperadius = 1;
```

```
Clear[d, slope];
d = 2 stovepiperadius;
slope = -z/x;
longaxis = Sqrt[d^2 + (d slope)^2];
{d, slope, N[longaxis]}
```

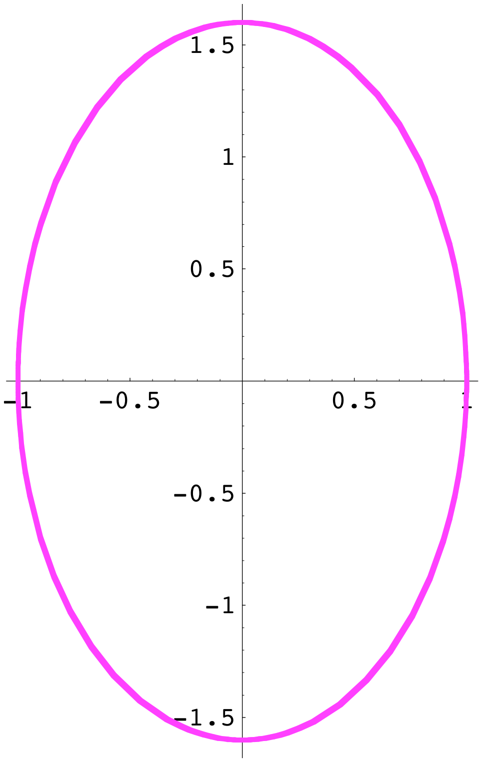
$\left\{2, -\frac{5}{4}, 3.20156\right\}$

MCB 10/21/07: So the long axis of the ellipse would be 3.20156 feet.

RC: 10/20/07: A graph of the final ellipse would be good, too.

MCB 10/21/07: Plotting the ellipse by itself is easy -- here it is

```
Clear[x, y, t];
{x[t_], y[t_]} = {Cos[t], longaxis/2 Sin[t]};
ellipseplot =
  ParametricPlot[{x[t], y[t]}, {t, 0, 2 Pi}, PlotStyle -> {{Thickness[0.01], Magenta}}];
```



But plotting it on the roof is a little trickier. You would need perpendicular frame unit vectors perpframe[1] and perpframe[1] in the plane of the roof, and you would need to know the 3-D coordinates of the center of the ellipse.

The center of the pipe as given above has {x, y} coordinates

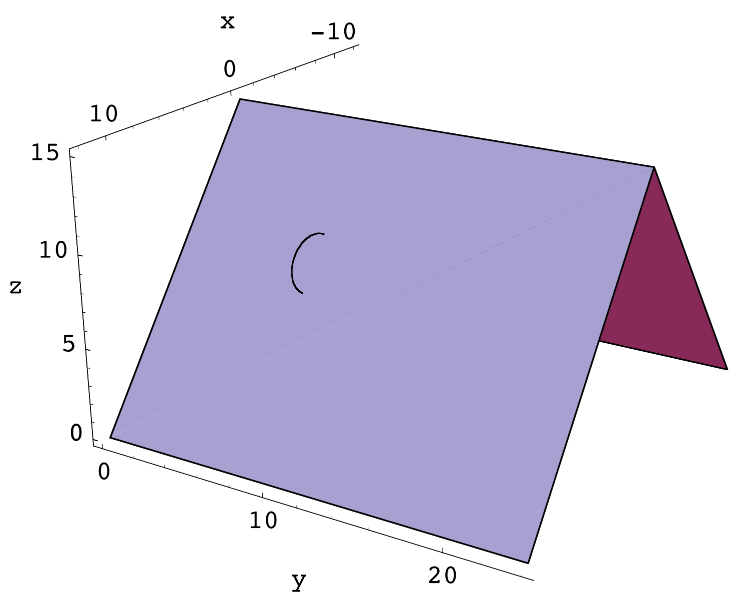
$$\text{xstovepipecenter} = 5$$
$$\text{ystovepipecenter} = 8$$

and the z coordinate should be

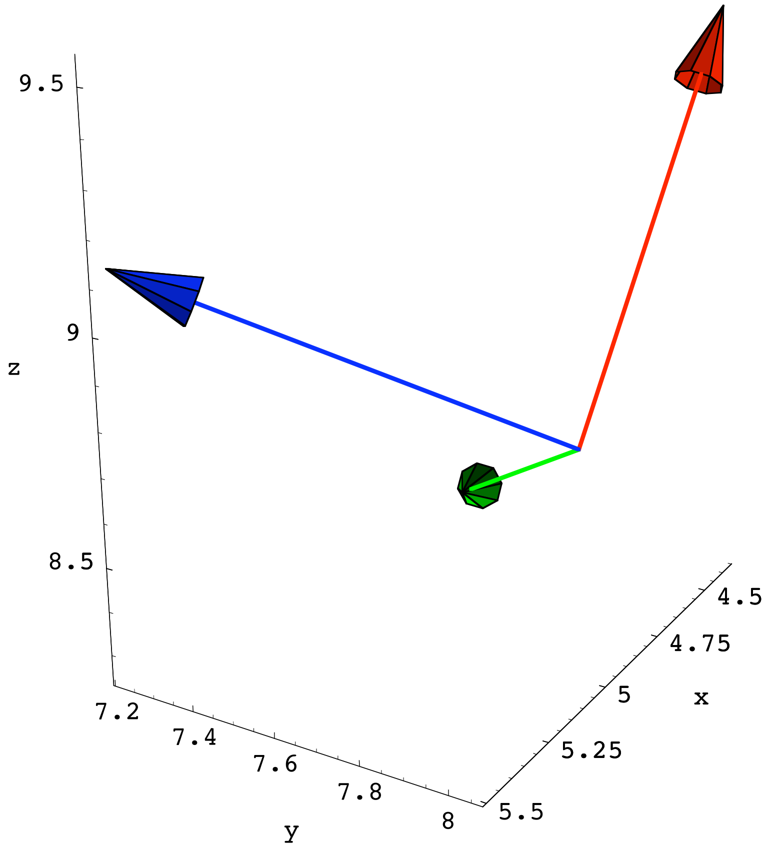
$$\text{zstovepipecenter} = \text{roofpeak } 15 - \left(\Delta z = \text{slope } \Delta x = \frac{5}{4}\, 5 = \frac{25}{4}\right)$$
$$\text{zstovepipecenter} = 15 - \frac{25}{4} = \frac{35}{4}$$

So the center of the ellipse and basepoint for the perpendicular frame vectors will be the point $\{5, 8, \frac{35}{4}\}$.

```
Clear[vector1, perpframe1];
vector1 = {0, 8, 15} - {5, 8, 35/4};
perpframe1 = vector1/Sqrt[vector1.vector1];
throwawayvector =
  {Random[Real, {-1, 1}], Random[Real, {-1, 1}], Random[Real, {-1, 1}]};
Y = throwawayvector × perpframe1;
perpframe3 = Y/Sqrt[Y.Y];
perpframe2 = perpframe3 × perpframe1;
centerpoint = {5, 8, 35/4};
hungellipseplot = ParametricPlot3D[centerpoint + x[t] perpframe3 + y[t] perpframe1 ,
   {t, 0, 2 Pi}, DisplayFunction -> Identity];
Show[roof[12, 24, 15], hungellipseplot, ViewPoint -> CMView, Axes -> True, Boxed -> False,
  PlotRange → All, AxesLabel -> {"x", "y", "z"}, DisplayFunction → $DisplayFunction];
```




```
Show[Arrow[perpframe1, Tail → centerpoint, VectorColor → Red],
  Arrow[perpframe2, Tail → centerpoint, VectorColor → Green],
  Arrow[perpframe3, Tail → centerpoint], ViewPoint -> CMView, Axes -> True, Boxed -> False,
  PlotRange → All, AxesLabel -> {"x", "y", "z"}, DisplayFunction → $DisplayFunction];
```



MCB 10/22/07: Hmmm. The vectors look OK but the hungellipseplot won't show more than half of the ellipse on the roof. What to do?

RC: 10/25/07: When you have two graphics objects that are close together, and they are not transparent/translucent, one will block the other. One way to show a curve like this is to boost the z-coordinate a little bit. Push it higher than than the roof by 0.1 or something, and then it won't get blocked by the plane.