Growth

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1.01 Growth

Give It a Try G2

Graphics Primitives

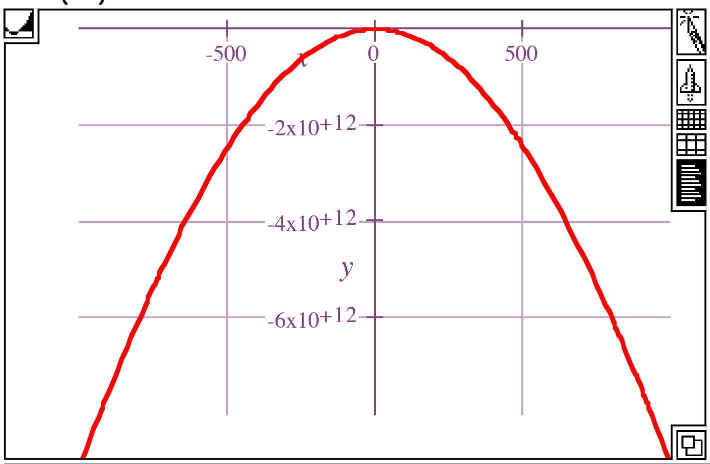
BR 1/27: Graphs edited.

G.2) Global scale*

G.2.a)

Look at:

$f(x) = x^4 - 10000000\,x^2$

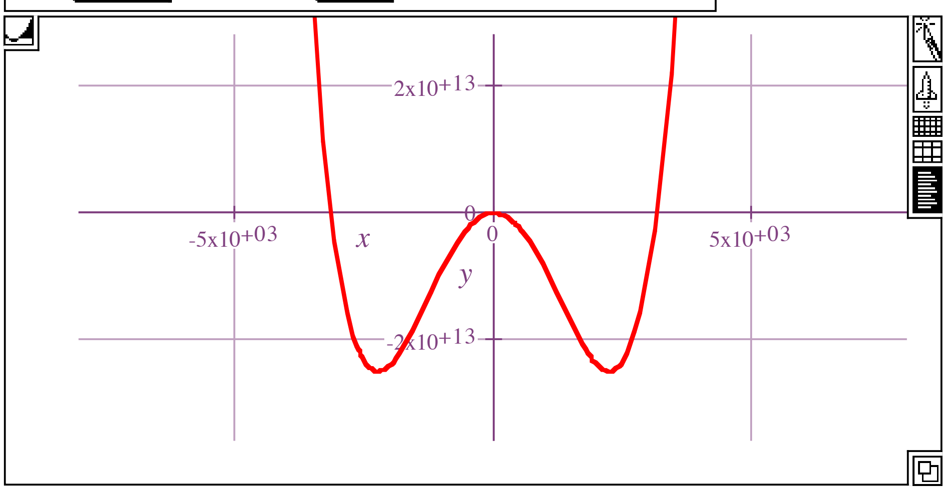


$-1000 \ldots 1000$ = left...right Stretch to Fit

$-8000000000000 \ldots 0$ = bottom...top cropping Moderately

Graph Building Blocks

Curve at $(x, f[x])$ where x = left ... right with a heavy line, colored Red.



– 8000 ... 8000 = left...right Stretch to Fit

– 36000000000000 ... 28000000000000 = bottom...top cropping Moderately

Graph Building Blocks

Curve at $(x, f[x])$ where x = left ... right with a heavy line, colored Red.

Is this a good global scale plot of

$$f(x) = x^4 - 10000000\,x^2 \;?$$

Why or why not?

If it is not a good global scale plot of $f(x)$, then give a good global scale plot of $f(x)$.

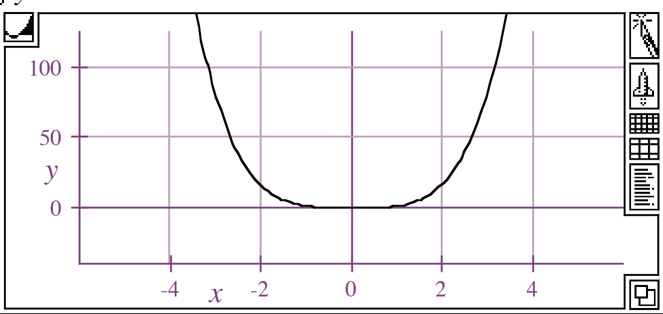
No, because the leading term (x^4) is not represented.

RC: 01/21/13: Sort of. Not the best explanation.

BR 1/22: This graph is not a good global scale plot of f(x) because it displays the non-leading term (-x^2) instead of the leading term of x^4, which upon zooming out, is shown to dominate the rest of the graph, giving a graph similar to the following graph.

RC: 1/26/13: Good

$y = x^4$



G.2.b)

Put

$$f(x) = \frac{2\,x^6 + 50\,x^2}{x^6 + 3\,x^2 + 1}.$$

What do you say are the limiting values

$$\lim_{x \to \infty} f(x)$$

and

$$\lim_{x \to -\infty} f(x)?$$

$$\frac{2x^6 + 50x^2}{x^6 + 3x^2 + 1}$$

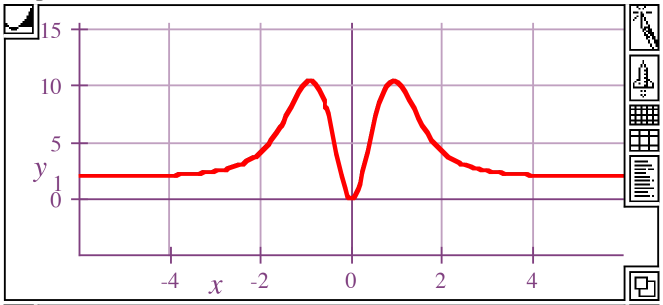
?

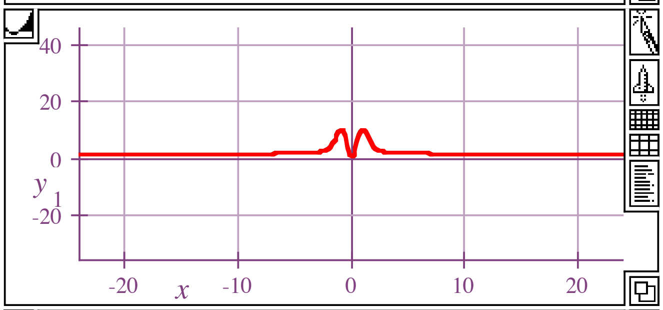
Same global scale behavior as:

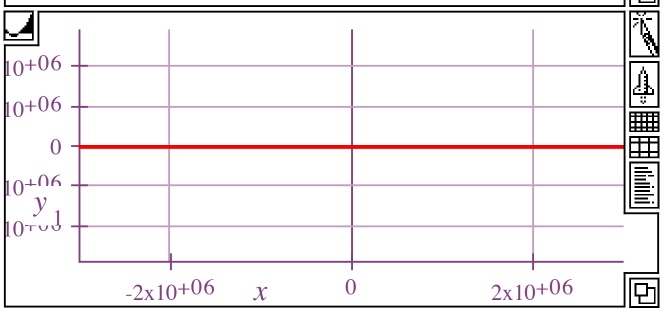
$$\frac{2x^6}{x^6} = 2$$

$$f(x) = \frac{2x^6 + 50x^2}{x^6 + 3x^2 + 1}$$

$y_1 = f(x)$





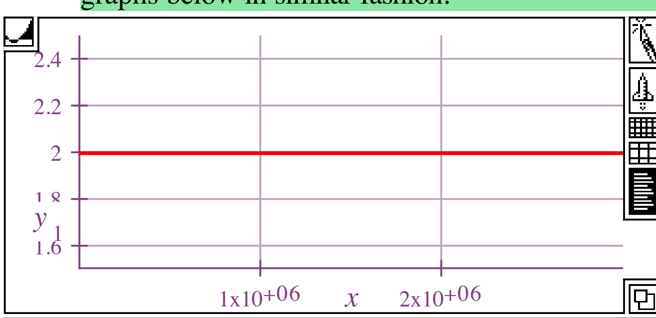


$y_2 = 2$

BR 1/22: The limiting value as the funciton approaches +/-∞ is +/- 2.

RC: 01/21/13: Your graphs are not convincing - x= 6 is not too near ∞. Also, do a dominant term analysis as well. Improve other problems similarly.

RC: 1/26/13: Here is the best graph. Fix your graphs below in similar fashion.



0 ... 3000000 = left...right Stretch to Fit

1.5 ... 2.5 = bottom...top cropping Moderately

Graph Building Blocks

Curve at (x, y_1) where x = left ... right with a normal line, colored Black.

Curve at (x, y_1) where x = left ... right with a heavy line, colored Red.

G.2.c)

What do you say is the limiting value

$$\lim_{x \to \infty} \frac{x^9 + 4e^{0.6x}}{3x^{12} + 2e^{0.6x}}?$$

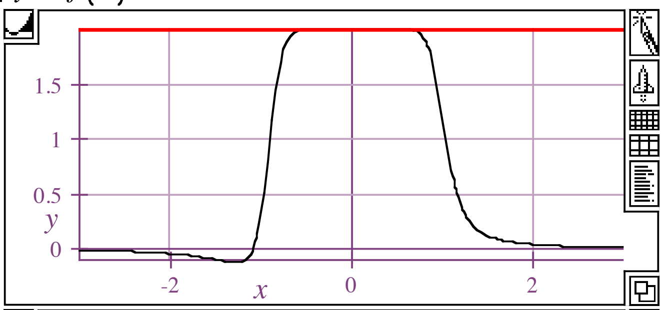
Illustrate with a plot.

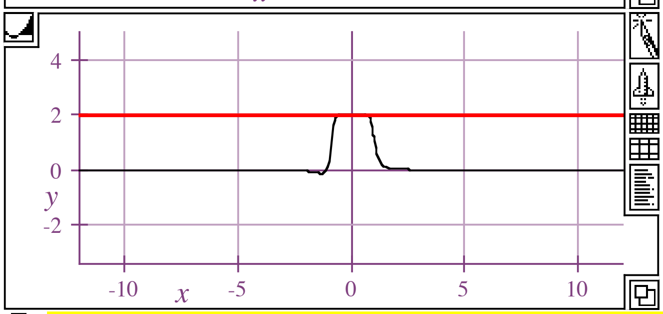
$$f(x) = \frac{x^9 + 4e^{0.6x}}{3x^{12} + 2e^{0.6x}}$$

$$\frac{x^9 + 4e^{0.6x}}{3x^{12} + 2e^{0.6x}}$$

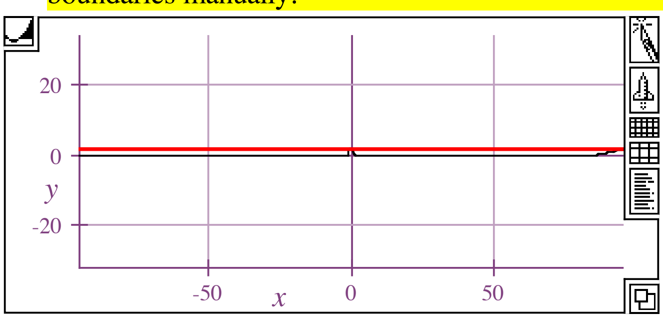
$$\frac{4e^{0.6x}}{2e^{0.6x}} = 2$$

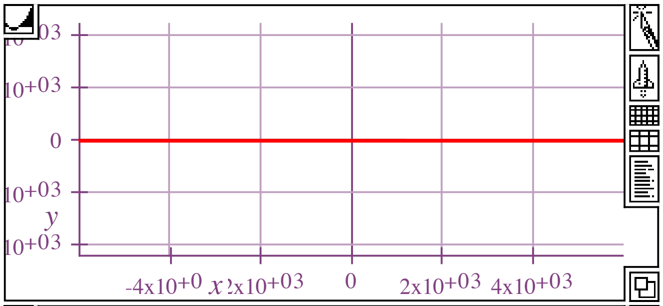
$y = f(x)$

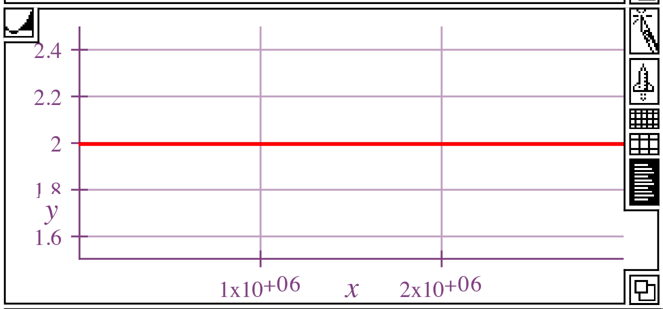




RC: 2/3/13:The y-scale on the graphs below are too large to see the intended value. You can 't just use the zoom out button, but rather you have to open up the details box and edit the window boundaries manually.







0 ... 3000000 = left...right Stretch to Fit

1.5 ... 2.5 = bottom...top cropping Moderately

Graph Building Blocks

Curve at (x, y) where x = left ... right with a normal line, colored Black.

Curve at (x, y_2) where x = left ... right with a heavy line, colored Red.

$y_2 = 2$

BR 1/22: The limiting value as the funciton approaches $+\infty$ is +2.

G.2.d)

What do you say is the limiting value

$$\lim_{x \to \infty} \frac{3x^8 - 123\cos(x) - 6x^2}{e^{0.4x}} \; ?$$

Illustrate with a plot.

$$f(x) = \frac{3x^8 - 123\cos(x) - 6x^2}{e^{0.4x}}$$

Leading term analysis:

$$\frac{3x^8 - 123\cos(x) - 6x^2}{e^{0.4x}}$$

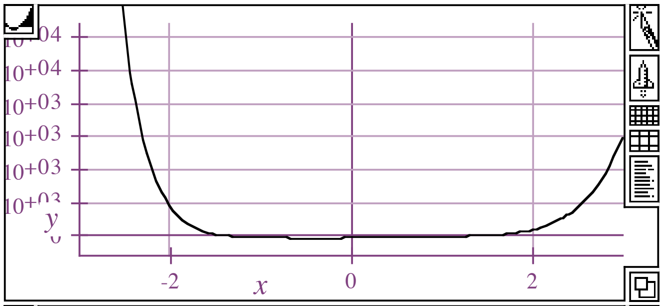
Leading term analysis:

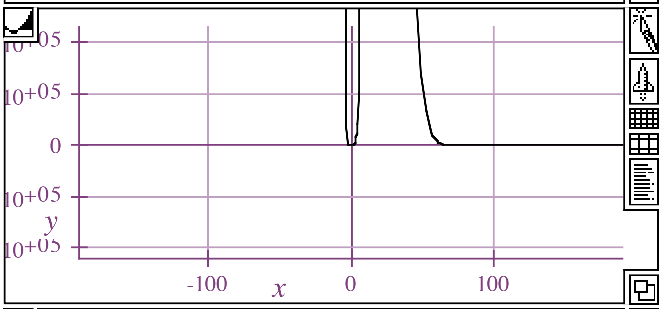
Same limiting behvaior as:

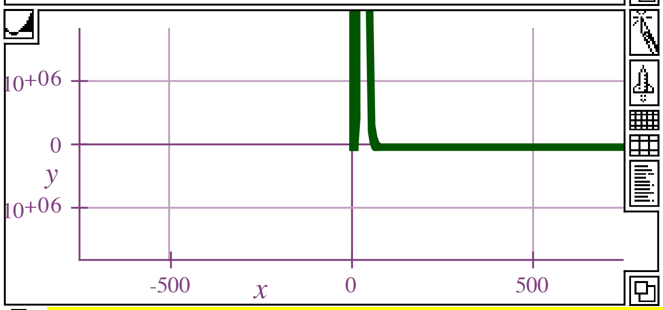
$$\frac{3x^8}{e^{0.4x}}$$

BR 1/22 The limiting value as the funciton approaches $+\infty$ is 0.

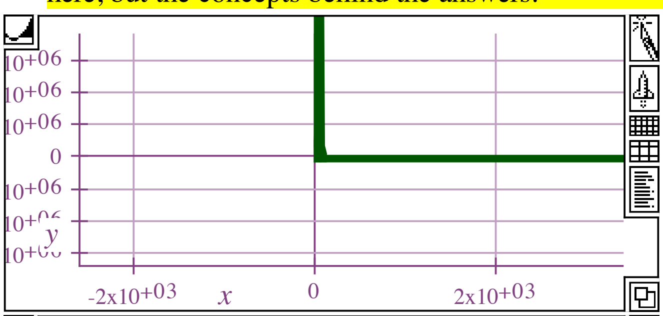
$y = f(x)$

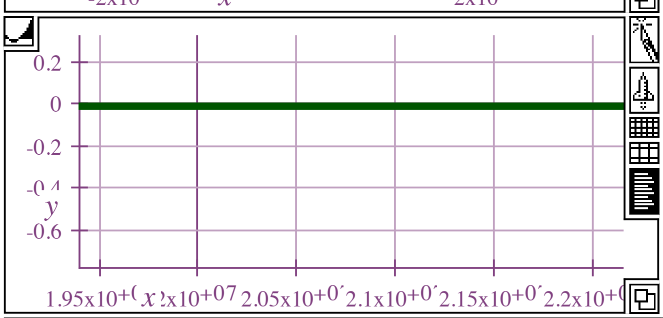






RC: 2/3/13: Same comment as previous, plus: Give some explanations on these. Pretend you are teaching me, a fellow student, not just the answers here, but the concepts behind the answers.





19400000 ... 22150000 = left...right Stretch to Fit

– 0.78 ... 0.32 = bottom...top cropping Moderately

Graph Building Blocks

Curve at (x, y) where x = left ... right with a extra heavy line, colored Dark Green.

G.2.e)

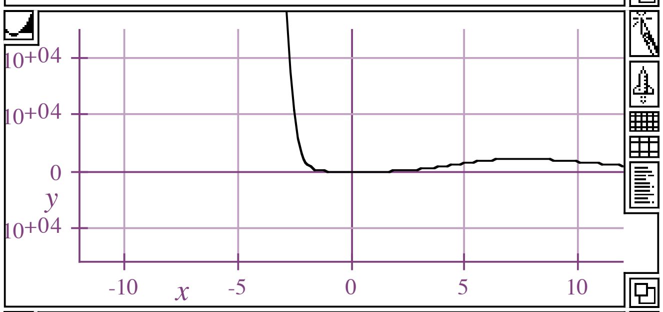
What do you say is the limiting value

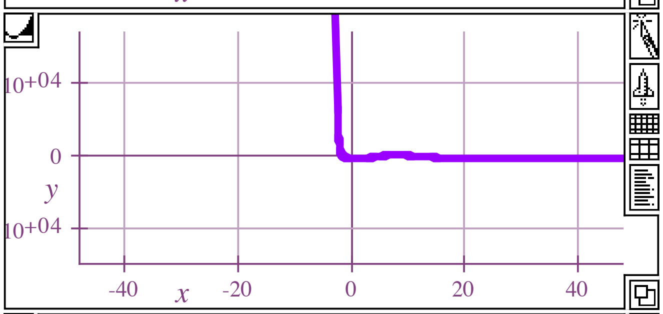
$$\lim_{x \to \infty} e^{-0.8x}\left(1+5\,x^{6}\right) ?$$

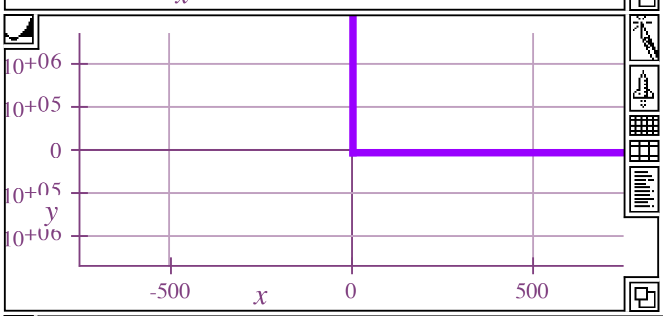
Illustrate with a plot.

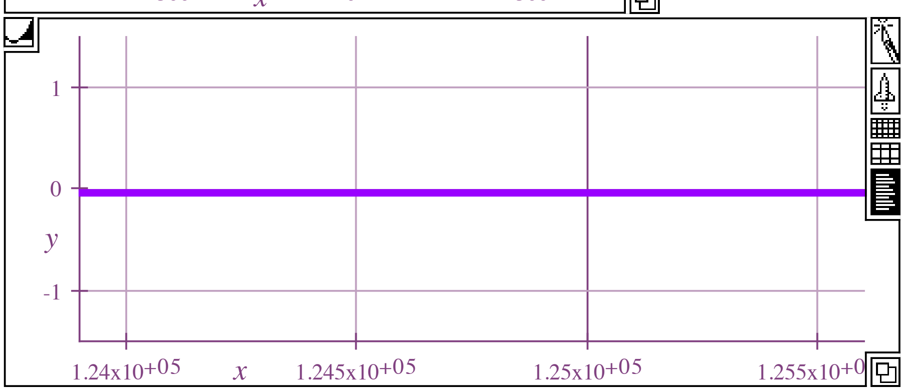
$f(x) = e^{-0.8x}\left(1+5x^{6}\right)$

$y = f(x)$









123900 ... 125600 = left...right   Stretch to Fit

− 1.5 ... 1.5 = bottom...top   cropping Moderately

Graph Building Blocks

Curve at (x, y) where x = left ... right with a extra heavy line, colored Purple.

BR 1/23 The limiting value as the funciton approaches +∞ is 0

G.2.f)

What do you say is the limiting value

$$\lim_{x \to \infty} \frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}}?$$

Illustrate with a plot.

$$\frac{\left(3e^{-x} - e^{-3x}\right)\left(3e^{-x}\right)}{\left(e^{-3x} + e^{-x}\right)\left(3e^{-x}\right)}$$

$$\frac{\left(3e^{-x} - e^{-3x}\right)\left(3e^{-x}\right)}{\left(e^{-3x} + e^{-x}\right)\left(3e^{-x}\right)} = \frac{9\left(e^{-x}\right)^2 - 3e^{\frac{1}{1}(-x)+\frac{1}{1}(-3x)}}{\left(e^{-3x} + e^{-x}\right)\left(3e^{-x}\right)} \quad \textit{Expand}$$

$$\frac{\left(3e^{-x} - e^{-3x}\right)\left(3e^{-x}\right)}{\left(e^{-3x} + e^{-x}\right)\left(3e^{-x}\right)} = \frac{-3e^{-x-3x} + 9e^{-2x}}{\left(e^{-3x} + e^{-x}\right)\left(3e^{-x}\right)} \quad \textit{Simplify}$$

$$\frac{\left(3e^{-x} - e^{-3x}\right)\left(3e^{-x}\right)}{\left(e^{-3x} + e^{-x}\right)\left(3e^{-x}\right)} = \frac{-3e^{-4x} + 9e^{-2x}}{\left(e^{-3x} + e^{-x}\right)\left(3e^{-x}\right)} \quad \textit{Simplify}$$

$$\frac{\left(3e^{-x} - e^{-3x}\right)\left(3e^{-x}\right)}{\left(e^{-3x} + e^{-x}\right)\left(3e^{-x}\right)} = \frac{-3e^{-4x} + 9e^{-2x}}{3\left(e^{-x}\right)^2 + 3e^{\frac{1}{1}(-x)+\frac{1}{1}(-3x)}} \quad \textit{Expand}$$

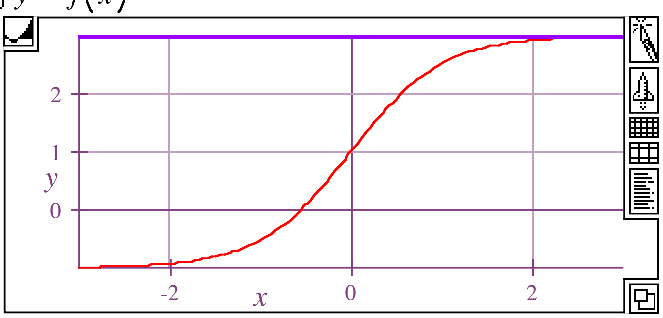
$$\frac{\left(3e^{-x} - e^{-3x}\right)\left(3e^{-x}\right)}{\left(e^{-3x} + e^{-x}\right)\left(3e^{-x}\right)} = \frac{-3e^{-4x} + 9e^{-2x}}{3e^{-x-3x} + 3e^{-2x}} \quad \textit{Simplify}$$

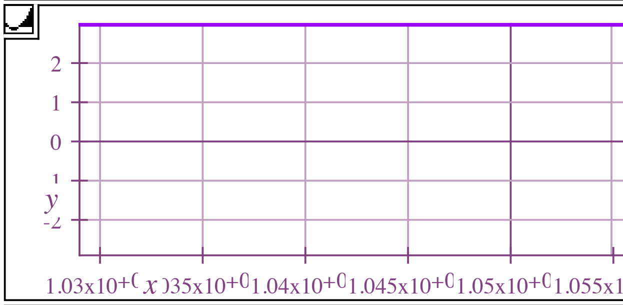
$$\frac{\left(3e^{-x} - e^{-3x}\right)\left(3e^{-x}\right)}{\left(e^{-3x} + e^{-x}\right)\left(3e^{-x}\right)} = \frac{-3e^{-4x} + 9e^{-2x}}{3e^{-4x} + 3e^{-2x}} \quad \textit{Simplify}$$

$$\frac{9e^{-2x}}{3e^{-2x}} = 3$$

$$f(x) = \frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}}$$

$$y = f(x)$$





102900 ... 105550 = left...right   Stretch to Fit

– 2.9 ... 3 = bottom...top   cropping Moderately

Graph Building Blocks

Curve at (x, y) where x = left ... right with a normal line, colored Red.

Curve at (x, y_2) where x = left ... right with a heavy line, colored Purple.

$y_2 = 3$

BR: 1/23 The limiting value as the funciton approaches +∞ is 3

Tip

G.2.g)

Rank the following functions in order of dominance as $x \to \infty$:

$$0.0001\ x^{24},\ 0.0004\ e^{0.01x},\ 89\ x^2,\ \sqrt{x},\ 17\ x,\ 0.08\ x^3, 0.0000013\ e^{2x},\ 100\ x^{0.4}.$$

$0.0000013\ e^{2x}$

$0.0004\ e^{0.01x}$

$0.0001\ x^{24}$

$0.08\ x^3$

$89\ x^2$

$17\ x$

$100\ x^{0.4}$

$\sqrt{x}$

G.2.h)

Plot

$$f(x) = \frac{2x^4 - 40x + 1}{x^2 + x + 12}$$

in global scale.

What simpler function mimicks the global scale behavior of $f(x)$?

Give a number b so that $f(x)$ is in its global scale behavior for $|x| > b$.

$$f(x) = \frac{2x^4 - 40x + 1}{x^2 + x + 12}$$

?

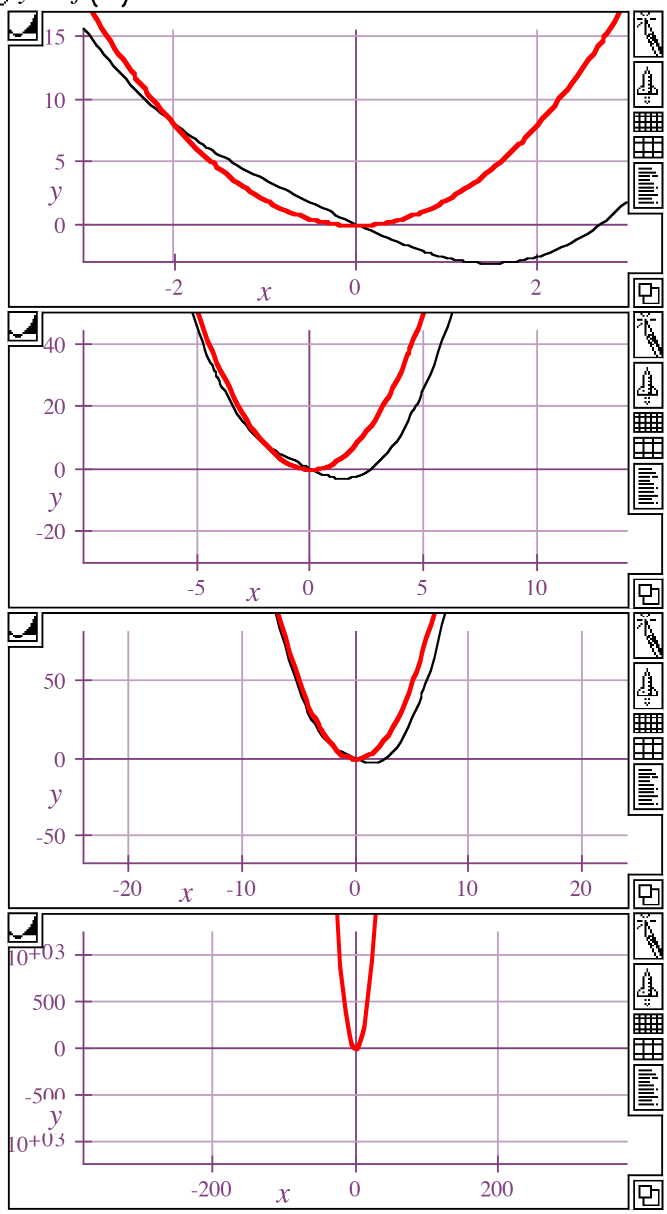
Dominant term analysis:

$$\frac{2x^4 - 40x + 1}{x^2 + x + 12}$$

$$\frac{2x^4}{x^2}$$

$$\frac{2x^4}{x^2} = 2x^2 \quad \textit{Simplify}$$

$y = f(x)$



$y_2 = 2x^2$

BR 1/22 b =5, at point ~| x|=5, the y values share the same distance from the focus on both sides of the parabola, denoting that the dominant behavior, that of 2x^2, has become prevalent.