Growth

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1.05 Using The Tools

Give It a Try G10

DD: 1/17/13: A couple to fix.

DD: 1/16/13: More to do.

DD: 1/15/13: A few to reconsider.

Graphics Primitives

This LiveMath Independence Declaration allows the derivatives to be computed as intended in this notebook.

The variables $(a, b, c, x, y, t, r, k, s, z, A, w, D, S, v)$ are independent of each other.

G.10) Other max-min problems

G.10.a)

Explain the statement:

Of all rectangles with a fixed perimeter, the square measures out to the largest area.

Tip:

The sides of the rectangle are x and y units long.

$\text{area}(x,y) = x\,y$

$2x + 2y = P$

where P is the given perimeter.

MR, 1/15: OK: Let Perimeter=P

$2x + 2y = P$

MR, 1/15: Solve for y

$2x + 2y = P$

$y = \frac{1}{2}(-2x + P)$ *Isolate*

MR, 1/15: Plug in to clear y from the function.

$\text{area}(x, y) = x\,y$

$f(x) = x\,y$

$f(x) = \frac{1}{2}x(-2x + P)$ *Substitute*

$f(x) = -x^2 + \frac{1}{2}P\,x$ *Expand*

MR, 1/15: Take the derivative to find the maximum:

$f'(x) = \frac{d}{dx}\left(-x^2 + \frac{1}{2}P\,x\right)$

$f'(x) = -2x + \frac{1}{2}P$ *Simplify*

$0 = -2x + \frac{1}{2}P$

$x = \frac{1}{4}P$ *Isolate*

MR, 1/15: This tells me we 're at a maximum when one of the sides is 1/4 the total perimter, or when we 've got a square.

DD: 1/16/13: Well, you should solve for y and show that it is also P/4.

MR, 1/17: OK: Let Perimeter=P

$2x + 2y = P$

MR, 1/15: Solve for x

$2x + 2y = P$

$x = \frac{1}{2}(-2y + P)$ *Isolate*

MR, 1/15: Plug in to clear x from the function.

$\text{area}(x, y) = x\,y$

$f(y) = x\,y$

$f(y) = \frac{1}{2}y(-2y + P)$ *Substitute*

$f(y) = -y^2 + \frac{1}{2}P\,y$ *Expand*

MR, 1/15: Take the derivative to find the maximum:

$f'(y) = \frac{d}{dy}\left(-y^2 + \frac{1}{2}P\,y\right)$

$f'(y) = -2y + \frac{1}{2}P$ *Simplify*

$0 = -2y + \frac{1}{2}P$

$y = \frac{1}{4}P$ *Isolate*

MR, 1/15: These two case theories tell me we 're at a maximum when one of the sides is 1/4 the total perimter, or when we 've got a square.

MR, 12/19: Let Perimeter P=16

DD: 1/15/13: Let 's not set P to a number. Let 's let P remain an unspecified constant. That 's the only way to solve the problem in all its generality. Please redo this.

$2x + 2y = 16$

MR, 12/19: Solve for y.

$y = \frac{1}{2}(-2x + 16)$ *Isolate*

$y = -x + 8$ *Expand*

MR, 12/19: Plug in to clear y from the function.

$\text{area}(x, y) = x\,y$

$f(x) = x\,y$

$f(x) = x(-x + 8)$ *Substitute*

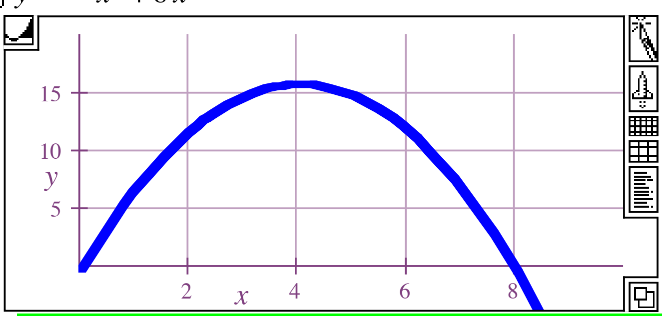
$f(x) = -x^2 + 8x$ *Expand*

$f(x) = -x^2 + 8x$

$f(x) = 7$ *Substitute*

$x = 7$

$y = -x^2 + 8x$



MR, 12/19: This plot represents where x is one side of a rectangle with perimeter 16 and f(x) is that rectangle 's area.

rectangle 's area.

MR, 12/19: Find the maximum using the derivative.

$f(x) = -x^2 + 8x$

$f'(x) = -2x + 8$

$0 = -2x + 8$

$x = 4$ *Isolate*

MR, 12/19: The maximum area occurs when x=4.
At this point, the other side is 4, which is a square.

? = ?