1.01 Growth
Give It a Try G2

Graphics Primitives

G.2) Global scale*

G.2.a)

Look at:

\[ f(x) = x^4 - 10000000 \times x^2 \]

N Ch 19.01.2013 A good global scale plot for this graph looks like \( f(x) = x^4 \)

RC: 01/21/13: Good

\[ y = x^4 \]

Graph Building Blocks

- Curve at \( (x, f(x)) \) where \( x = \) left ... right with a heavy line, colored Red.
- Curve at \( (x, y) \) where \( x = \) left ... right with a heavy line, colored Purple.

Is this a good global scale plot of \( f(x) = x^4 - 10000000 \times x^2 \) ?

Why or why not?
If it is not a good global scale plot of $f(x)$, then give a good global scale plot of $f(x)$.

\[ f(x) = x^4 - 10000000 x^2 \]
\[ y = x^4 \]

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- $-42000 \ldots 42000 = \text{left...right}$
  - Stretch to Fit

- $-1.7 \times 10^{16} \ldots 1.7 \times 10^{16} = \text{bottom...top}$
  - Moderately

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Graph Building Blocks

- Curve at $(x, f(x))$ where $x = \text{left ... right}$ with a heavy line, colored Red.
- Curve at $(x, y)$ where $x = \text{left ... right}$ with a dashed line, colored Blue.

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**G.2.b)**

Put

\[ f(x) = \frac{2 x^6 + 50 x^2}{x^6 + 3 x^2 + 1} \]

What do you say are the limiting values

\[ \lim_{x \to \infty} f(x) = 2 \]

and

\[ \lim_{x \to -\infty} f(x) = 2 \]

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\[ f(x) = \frac{2 x^6 + 50 x^2}{x^6 + 3 x^2 + 1} \]

\[ \frac{2 x^6}{x^6} \]

\[ \frac{2 x^6}{x^6} = 2 \quad \text{Simplify} \]

\[ y = f(x) \]
Graph Building Blocks

Curve at \((x, y)\) where \(x = \text{left} \ldots \text{right}\) with a normal line, colored Black.

RC: 01/21/13: Your y-axis scale is too large to show \(y=2\) as the limiting value. Your scale should be something like 1.5...2.5

N Ch 22.01.2013 Fixed it.

RC: 01/22/13: Good

G.2.e) What do you say is the limiting value

\[
\lim_{x \to \infty} \frac{x^9 + 4e^{0.6x}}{3x^{12} + 2e^{0.6x}} = 2
\]

Illustrate with a plot.

\[
f(x) = \frac{x^9 + 4e^{0.6x}}{3x^{12} + 2e^{0.6x}}
\]

RC: 01/22/13: good

\[y = f(x)\]
N Ch 19.01.2013 "exponent part" doesn't depend on x

RC: 01/21/13: Incorrect. Go out farther to the right on your graph. What do your computations below have to do with this graph?

4 \frac{e^{0.6x}}{2 \ e^{0.6x}}

\[ \frac{4 e^{0.6x}}{2 e^{0.6x}} = 2 \text{ Simplify} \]

RC: 01/22/13: You don't need the "power part" since the exponentials dominate.

"Power part"

9 \ \frac{x}{3 \ x^2}

\[ \frac{9 x}{3 x^2} = \frac{1}{3 x^3} \text{ Simplify} \]

\[ \lim_{x \to \infty} \frac{1}{3 x^3} = 0 \]

N Ch 22.01.2013 Thank God! I was really uncomfortable with that zero limit. My problem is I'm terrified of limits. I'm still not sure what they are.

So, for small x, f(x) tends to 0

50^9 + 4 e^{0.6\cdot50}

\[ \frac{50^9 + 4 e^{0.6\cdot50}}{3 \cdot 50^2 + 2 e^{0.6\cdot50}} = 2.72502898699495 \times 10^{-6} \text{ Calculate} \]

then, for large x, it goes up to 2

100^9 + 4 e^{0.6\cdot100}

\[ \frac{100^9 + 4 e^{0.6\cdot100}}{3 \cdot 100^2 + 2 e^{0.6\cdot100}} = 1.97407104287832 \text{ Calculate} \]

Am I right?

G.2.d)

What do you say is the limiting value

\[ \lim_{x \to \infty} \frac{3 x^8 - 123 \cos(x) - 6 x^2}{e^{0.4x}} = 0 \]

Illustrate with a plot.

\[ f(\ x) = \frac{3 \ x^8 - 123 \cos\left(\ x\right) - 6 \ x^2}{e^{0.4\ x}} \]
Graph Building Blocks

Curve at \((x, y)\) where \(x = \text{left} \ldots \text{right}\) with a heavy line, colored \text{Red}

\[y = f(x)\]

\[y = f(x)\]

80 \ldots 1080 = \text{left} \ldots \text{right} \quad \text{Stretch to Fit}

\(-1 \ldots 1 = \text{bottom} \ldots \text{top} \quad \text{cropping Moderately}\)

G.2.e)

What do you say is the limiting value

\[
\lim_{x \to \infty} e^{-0.8x\left(1 + 5x^6\right)} = 0
\]

Illustrate with a plot.

\[y = f(x)\]

\[y = f(x)\]

N Ch 19.01.2013 \(e^{-0.8x\left(1 + 5x^6\right)} = \frac{1}{e^{0.8x}}\)

RC: 01/21/13: Your y-axis scale is too large to show \(y=0\) as the limiting value.

RC: 01/22/13: Good

N Ch 22.01.2013 Fixed it.
What do you say is the limiting value
\[
\lim_{x \to \infty} \frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}} = 3
\]
Illustrate with a plot.
G.2.g) Rank the following functions in order of dominance as $x \to \infty$:

$$0.0001 \cdot x^{24},\ 0.0004 \cdot e^{0.01x},\ 89 \cdot x^2,\ \sqrt{x},\ 17 \cdot x,\ 0.08 \cdot x^3,\ 0.0000013 \cdot e^{2x},\ 100 \cdot x^{0.4}.$$ 

N Ch 19.01.2013 - 0.0000013 $e^{2x},\ 0.0004 \cdot e^{0.01x},\ 0.0001 \cdot x^{24},\ 0.08 \cdot x^3,\ 89 \cdot x^2,\ \sqrt{x},\ 17 \cdot x,\ 100 \cdot x^{0.4}$

RC: 01/21/13: Good

G.2.h) Plot

$$f(x) = \frac{2 \cdot x^4 - 40 \cdot x + 1}{x^2 + x + 12}$$

in global scale.

What simpler function mimicks the global scale behavior of $f(x)$?

Give a number $b$ so that $f(x)$ is in its global scale behavior for $|x| > b$.

RC: 01/21/13: $b=2$ is probably sufficient.

N Ch 22.01.2013 May I choose $b=8$? Or $b=2$ is enough?

RC: 01/22/13: You may choose any number past $b=1$ or so. The problem is a little vague (on purpose)

$y = f'(x)$

Graph Building Blocks

Curve at $(x, y)$ where $x = \text{left ... right}$ with a [normal] line, colored [Blue].
Graph Building Blocks

- Curve at \((x, y)\) where \(x = \text{left} \ldots \text{right}\) with a \text{normal line, colored Blue}.
- Curve at \((x, y)\) where \(x = \text{left} \ldots \text{right}\) with a \text{dashed line, colored Red}.

\[y = 2x^2\]
\[b = 8\]