



Approximation

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3.01 Splines

Give It a Try G2

LS: 8/24/12: See comment below on 2.d.i, then do similarly on 2.d.ii

LS: 8/22/12: See hint on 2.d.i. Need to fix up 2.d.i and 2.d.ii

LS: 8/21/12: I didn't see any changes below.
Wrong file?

8/22/12: Almost right. This is what I wrote:

8/19/12: So $a_1 = 0$ and $a_0 = 0$... Does that mean that $f(x) = 0$ as well? I don't appear to be understanding this.... My math is all funky.

LS: 8/17/12: See hints below on 2.d.i and 2.d.ii

Experience with the starred problems will be especially beneficial for understanding later lessons.

G.2) Splining functions and polynomials*

G.2.a.i) Splining with $\sin(x)$

Find the polynomial of degree 3 that has the highest possible order of contact with $f(x) = \sin(x)$ at $x = 0$.

Plot the spline knotted at $\{0,0\}$ with $f(x)$ on the right and your polynomial on the left

Also plot on the same axes $f(x)$, the polynomial, and the spline for $-2 \leq x \leq 2$.

Describe what you see.

<input type="checkbox"/>	$f(x) = \sin(x)$
<input type="checkbox"/>	$g(x) = \sum_{k=0}^3 a_k x^k$
<input checked="" type="checkbox"/>	$g(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$ Expand

<input type="checkbox"/>	$f(x) = \sin(x)$
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$$\square g(x) = \sum_{k=0}^3 a_k x^k$$

$$\triangle g(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$\square f(0) = g(0)$$

$$\triangle \sin(0) = a_3 0^3 + a_2 0^2 + a_1 0 + a_0 \quad \text{Substitute}$$

$$\triangle 0 = a_0 \quad \text{Simplify}$$



$$\square f(x) = \sin(x)$$

$$\square g(x) = \sum_{k=0}^3 a_k x^k$$

$$\triangle g(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$\square f'(0) = g'(0)$$

$$\triangle \left. \frac{d}{dx} f(x) \right|_{x=0} = \left. \frac{d}{dx} g(x) \right|_{x=0} \quad \text{Simplify}$$

$$\triangle \left. \frac{d}{dx} \sin(x) \right|_{x=0} = \left. \frac{d}{dx} (a_3 x^3 + a_2 x^2 + a_1 x + a_0) \right|_{x=0} \quad \text{Substitute}$$

$$\triangle \cos(0) = 3 a_3 \cdot 0^2 + 2 a_2 \cdot 0 + a_1 \quad \text{Simplify}$$

$$\triangle 1 = a_1 \quad \text{Simplify}$$

$$\square f''(0) = g''(0)$$

$$\triangle \left. \frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right|_{x=0} = \left. \frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right|_{x=0} \quad \text{Simplify}$$

$$\triangle \left. \frac{d}{dx} \left(\frac{d}{dx} \sin(x) \right) \right|_{x=0} = \left. \frac{d}{dx} \left(\frac{d}{dx} (a_3 x^3 + a_2 x^2 + a_1 x + a_0) \right) \right|_{x=0} \quad \text{Subst}$$

$$\triangle -\sin(0) = 6 a_3 \cdot 0 + 2 a_2 \quad \text{Simplify}$$

$$\triangle 0 = 2 a_2 \quad \text{Simplify}$$

$$\square f'''(0) = g'''(0)$$

$$\triangle \left. \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right) \right|_{x=0} = \left. \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right) \right|_{x=0} \quad \text{Simplify}$$

$$\triangle \left. \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \sin(x) \right) \right) \right|_{x=0} = \left. \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} (a_3 x^3 + a_2 x^2 + a_1 x + a_0) \right) \right) \right|_{x=0}$$

$$\triangle -\cos(0) = 6 a_3 \quad \text{Simplify}$$

$$\triangle -1 = 6 a_3 \quad \text{Simplify}$$



$$\square a_0 = 0$$

$$\square a_1 = 1$$

$$\square 2 a_2 = 0$$

$$\triangle a_2 = \frac{1}{2} \cdot 0 \quad \text{Isolate}$$

$$\triangle a_2 = 0 \quad \text{Simplify}$$

$$\square 6 a_3 = -1$$

$$\triangle a_3 = \frac{1}{6}(-1) \quad Isolate$$

$$\triangle a_3 = -\frac{1}{6} \quad Simplify$$

$$\square g(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$\triangle g(x) = \left(-\frac{1}{6}\right)x^3 + 0x^2 + 1x + 0 \quad Substitute$$

$$\triangle g(x) = -\frac{1}{6}x^3 + x \quad Simplify$$



$$\square f(x) = \sin(x)$$

$$\square g(x) = -\frac{1}{6}x^3 + x$$

$$\square (f[0], g[0])$$

$$\triangle (f[0], g[0]) = \left(\sin[0], -\frac{1}{6} \cdot 0^3 + 0 \right) \quad Substitute$$

$$\triangle (f[0], g[0]) = (0, 0) \quad Simplify$$

$$\square (f'[0], g'[0])$$

$$\triangle (f'[0], g'[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} f(x) \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} g(x) \\ x=0 \end{array} \right] \right) \quad Simplify$$

$$\triangle (f'[0], g'[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \sin(x) \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ -\frac{1}{6}x^3 + x \right\} \\ x=0 \end{array} \right] \right) \quad Substitute$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} f(x) \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} g(x) \\ x=0 \end{array} \right] \right) = \left(\cos[0], -\frac{1}{2} \cdot 0^2 + 1 \right) \quad Simplify$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} f(x) \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} g(x) \\ x=0 \end{array} \right] \right) = (1, 1) \quad Simplify$$

$$\square (f''[0], g''[0])$$

$$\triangle (f''[0], g''[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \\ x=0 \end{array} \right] \right) \quad Simplify$$

$$\triangle (f''[0], g''[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \sin(x) \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left(-\frac{1}{6}x^3 + x \right) \right\} \\ x=0 \end{array} \right] \right)$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \\ x=0 \end{array} \right] \right) = (-\sin[0], -0) \quad Simplify$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} \frac{d}{dx} f(x) \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \frac{d}{dx} g(x) \\ x=0 \end{array} \right] \right) = (0, 0) \quad Simplify$$

$$\square (f'''[0], g'''[0])$$

$$\triangle (f'''[0], g'''[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \right\} \\ x=0 \end{array} \right] \right)$$

$$\triangle (f'''[0], g'''[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \sin(x) \right\} \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \left(-\frac{1}{6}x^3 + x \right) \right\} \right\} \\ x=0 \end{array} \right] \right)$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \right\} \\ x=0 \end{array} \right] \right) = (-\cos[0], -)$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f(x) \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g(x) \\ x=0 \end{array} \right] \right) = (-1, -1) \quad Simplify$$

$$\square (f''''[0], g''''[0])$$

$$\triangle(f''''[0], g''''[0]) = \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f(x) \right] \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g(x) \right] \right) \right\} \end{array} \right] \right)$$

$$\triangle(f''''[0], g''''[0]) = \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \sin(x) \right] \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g(x) \right] \right) \right\} \end{array} \right] \right)$$

$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right\} \end{array} \right] \right) =$$

$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f(x) \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g(x) \end{array} \right] \right) = (0,0) \quad \text{Si}$$

$(f''''[0], g''''[0])$

$$\triangle(f''''[0], g''''[0]) = \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f(x) \right] \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g(x) \right] \right) \right\} \end{array} \right] \right)$$

$$\triangle(f''''[0], g''''[0]) = \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \sin(x) \right] \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g(x) \right] \right) \right\} \end{array} \right] \right)$$

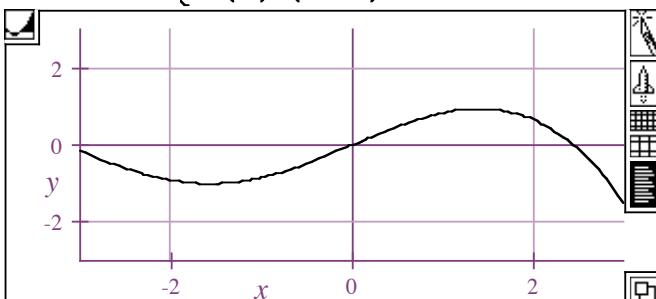
$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right\} \end{array} \right] \right) =$$

$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f(x) \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g(x) \end{array} \right] \right) = (1,1)$$

 As shown above, the order of contact between $f(x)$ and the polynomial at $x = 0$ is 4. Next, the graph of the two functions.

 RC: 7/31/12: Point of contact of order 3. $g(x)$ has only 3 degrees to contribute.

- $f(x) = \sin(x)$
- $g(x) = -\frac{1}{6}x^3 + x$
- $\text{spline}(x) = \begin{cases} f(x) & (x < 0) \\ g(x) & (x \geq 0) \end{cases}$

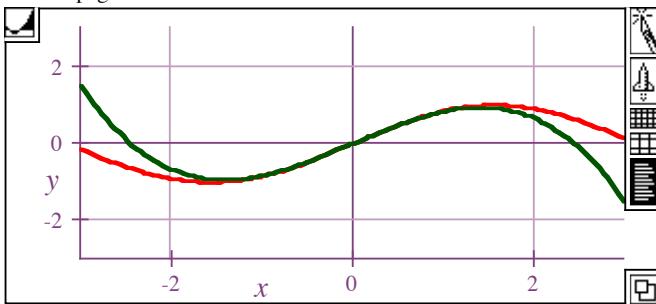


-3 ... 3 = left...right

-3 ... 3 = bottom...top cropping

 Graph Building Blocks

↳ Curve at $(x, \text{spline}[x])$ where $x = \text{left ... right}$
with a line, colored .



$-3 \dots 3 = \text{left...right}$ Stretch to Fit

$-3 \dots 3 = \text{bottom...top}$ cropping Moderately

Graph Building Blocks

Curve at $(x, f[x])$ where $x = \text{left ... right}$ with a heavy line, colored Red.

Curve at $(x, g[x])$ where $x = \text{left ... right}$ with a heavy line, colored Dark Green.

G.2.a.ii)

- Find the polynomial of degree 5 that has the highest possible order of contact with $f(x) = \sin(x)$ at $x = 0$.

Plot the spline knotted at $\{0,0\}$ with $f(x)$ on the right and your polynomial on the left. Also plot on the same axes $f(x)$, the polynomial, and the spline for $-2 \leq x \leq 2$.

Describe what you see, paying special attention to a comparison of the plots in parts



$f(x) = \sin(x)$

$g(x) = \sum_{k=0}^5 a_k x^k$

$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$ Expand



$f(x) = \sin(x)$

$g(x) = \sum_{k=0}^5 a_k x^k$

$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$ Expand

$f(0) = g(0)$

$\triangle \sin(0) = a_0 \cdot 0^0 + a_1 \cdot 0^1 + a_2 \cdot 0^2 + a_3 \cdot 0^3 + a_4 \cdot 0^4 + a_5 \cdot 0^5$ Substitute

$\triangle 0 = a_0$ Simplify



$$\square f(x) = \sin(x)$$

$$\square g(x) = \sum_{k=0}^5 a_k x^k$$

$$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 \quad \text{Expand}$$

$$\square f'(0) = g'(0)$$

$$\triangle_{x=0} \left[\frac{d}{dx} f(x) \right] = \underset{x=0}{\left[\frac{d}{dx} g(x) \right]} \quad \text{Simplify}$$

$$\triangle_{x=0} \left[\frac{d}{dx} \sin(x) \right] = \underset{x=0}{\left[\frac{d}{dx} (a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5) \right]}$$

$$\triangle \cos(0) = 5a_5 \cdot 0^4 + 4a_4 \cdot 0^3 + 3a_3 \cdot 0^2 + 2a_2 \cdot 0 + a_1 \quad \text{Simplify}$$

$$\triangle 1 = a_1 \quad \text{Simplify}$$

$$\square f''(0) = g''(0)$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right] = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right]} \quad \text{Simplify}$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \sin(x) \right) \right] = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} (a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5) \right) \right]}$$

$$\triangle -\sin(0) = 20a_5 \cdot 0^3 + 12a_4 \cdot 0^2 + 6a_3 \cdot 0 + 2a_2 \quad \text{Simplify}$$

$$\triangle 0 = 2a_2 \quad \text{Simplify}$$

$$\square f'''(0) = g'''(0)$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right) \right] = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right) \right]} \quad \text{Simplify}$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \sin(x) \right) \right) \right] = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} (a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5) \right) \right) \right]}$$

$$\triangle -\cos(0) = 60a_5 \cdot 0^2 + 24a_4 \cdot 0 + 6a_3 \quad \text{Simplify}$$

$$\triangle -1 = 6a_3 \quad \text{Simplify}$$

$$\square f''''(0) = g''''(0)$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right) \right) \right] = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right) \right) \right]} \quad \text{Simplify}$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \sin(x) \right) \right) \right) \right] = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} (a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5) \right) \right) \right) \right]}$$

$$\triangle \sin(0) = 120a_5 \cdot 0 + 24a_4 \quad \text{Simplify}$$

$$\triangle 0 = 24a_4 \quad \text{Simplify}$$

$$\square f''''''(0) = g''''''(0)$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right) \right) \right) \right] = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right) \right) \right) \right]}$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \sin(x) \right) \right) \right) \right) \right] = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} (a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5) \right) \right) \right) \right) \right]}$$

$$\triangle \cos(0) = 120a_5 \quad \text{Simplify}$$

$$\triangle 1 = 120a_5 \quad \text{Simplify}$$

$$\square a_0 = 0$$

$a_1 = 1$

$2a_2 = 0$

$\triangle a_2 = \frac{1}{2} \cdot 0 \quad Isolate$

$\triangle a_2 = 0 \quad Simplify$

$6a_3 = -1$

$\triangle a_3 = \frac{1}{6}(-1) \quad Isolate$

$\triangle a_3 = -\frac{1}{6} \quad Simplify$

$24a_4 = 0$

$\triangle a_4 = \frac{1}{24} \cdot 0 \quad Isolate$

$\triangle a_4 = 0 \quad Simplify$

$120a_5 = 1$

$\triangle a_5 = \frac{1}{120} \cdot 1 \quad Isolate$

$\triangle a_5 = \frac{1}{120} \quad Simplify$

$g(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$

$\triangle g(x) = 0x^0 + 1x^1 + 0x^2 + \left(-\frac{1}{6}\right)x^3 + 0x^4 + \frac{1}{120}x^5 \quad Substitute$

$\triangle g(x) = -\frac{1}{6}x^3 + \frac{1}{120}x^5 + x \quad Simplify$



$f(x) = \sin(x)$

$g(x) = -\frac{1}{6}x^3 + \frac{1}{120}x^5 + x$

$(f[0], g[0])$

$\triangle (f[0], g[0]) = \left(\sin[0], -\frac{1}{6}0^3 + \frac{1}{120}0^5 + 0 \right) \quad Substitute$

$\triangle (f[0], g[0]) = (0, 0) \quad Simplify$

$(f'[0], g'[0])$

$\triangle (f'[0], g'[0]) = \left(\left[x=0 \left[\frac{d}{dx} f(x) \right] \right], \left[x=0 \left[\frac{d}{dx} g(x) \right] \right] \right) \quad Simplify$

$\triangle (f'[0], g'[0]) = \left(\left[x=0 \left[\frac{d}{dx} \sin(x) \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ -\frac{1}{6}x^3 + \frac{1}{120}x^5 + x \right\} \right] \right] \right) \quad Subs$

$\triangle \left(\left[x=0 \left[\frac{d}{dx} f(x) \right] \right], \left[x=0 \left[\frac{d}{dx} g(x) \right] \right] \right) = \left(\cos[0], \frac{1}{24} \cdot 0^4 - \frac{1}{2} \cdot 0^2 + 1 \right) \quad Simplify$

$\triangle \left(\left[x=0 \left[\frac{d}{dx} f(x) \right] \right], \left[x=0 \left[\frac{d}{dx} g(x) \right] \right] \right) = (1, 1) \quad Simplify$

$(f''[0], g''[0])$

$\triangle (f''[0], g''[0]) = \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \right] \right] \right) \quad Simplify$

$\triangle (f''[0], g''[0]) = \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \sin(x) \right\} \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(-\frac{1}{6}x^3 + \frac{1}{120}x^5 + x \right) \right\} \right] \right] \right)$

$\triangle \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \right] \right] \right) = (-\sin[0], \frac{1}{6} \cdot 0^3 - 0) \quad Sii$

$$\square(f'''[0], g'''[0]) \quad \triangle \left(\left[x=0 \left[\frac{d}{dx} \frac{d}{dx} f\{x\} \right] \right], \left[x=0 \left[\frac{d}{dx} \frac{d}{dx} g\{x\} \right] \right] \right) = (0,0) \quad Simplify$$

$$\triangle(f'''[0], g'''[0]) = \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} g[x] \right) \right\} \right] \right]$$

$$\triangle(f'''[0], g'''[0]) = \left(\left[x=0 \right] \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \sin[x] \right) \right\}, \left[x=0 \right] \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[-\frac{1}{6}x^6 \right] \right) \right\} \right)$$

$$\triangle \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} g[x] \right) \right\} \right] \right] = \left(-\cos[0], \frac{1}{2} \right)$$

$$\triangle \left(\left[x=0 \left[\frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f\{x\} \right] \right], \left[x=0 \left[\frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g\{x\} \right] \right] \right) = (-1, -1) \quad Simplified$$

$$\square(f''''[0], g''''[0])$$

$$\triangle(f''''[0], g''''[0]) = \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f[x] \right] \right) \right\} \right]_{x=0}, \quad \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g[x] \right] \right) \right\} \right]_{x=0}$$

$$\triangle(f''''[0], g''''[0]) = \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \sin \{x\} \right] \right) \right\} \right]_{x=0}, \quad \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \cos \{x\} \right] \right) \right\} \right]_{x=0}$$

$$\triangle \left[\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f(x) \right] \right) \right\} \right|_{x=0}, \left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g(x) \right] \right) \right\} \right|_{x=0} \right] =$$

$$\triangle \left(\left[x=0 \left[\frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f\{x\} \right] \right], \left[x=0 \left[\frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g\{x\} \right] \right] \right) = (0,0) \quad \text{Si}$$

$$\square(f''''[0], g''''[0])$$

$$\triangle(f''''[0], g''''[0]) = \left(\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \right) \right\}, \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \right) \right\} \right),$$

$$\triangle(f''''[0], g''''[0]) = \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \sin(x) \right\} \right] \right) \right\} \right]_{x=0},$$

$$\triangle \left(\left[_{x=0} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \right] \right\} \right] \right], \left[_{x=0} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} g \right\} \right] \right\} \right] \right] \right]$$

$$\triangle \left[\left. x = 0 \right| \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f\{x\} \right], \left[\left. x = 0 \right| \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g\{x\} \right] = (1)$$

$$\square(f'''''[0], g'''''[0])$$

$$\triangle(f''''[0], g''''[0]) = \left(\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right\} \right\} \right\} \right|_{x=0}, \left. \frac{d}{dx} \right|_{x=0}$$

$$\triangle(f''''[0], g''''[0]) = \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \sin[x] \right) \right\} \right] \right) \right\} \right]_{x=0},$$

$$\triangle \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right] \right\} \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right] \right\} \right] \right] \right)$$

$$\square(f''''''[0], g''''''[0])$$

$$\begin{aligned} \triangle(f''''''[0], g''''''[0]) &= \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \sin \{x\} \right] \right) \right] \right) \right] \right] \right) \right) \\ \triangle \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f\{x\} \right] \right) \right] \right] \right] \right] \right), \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g\{x\} \right] \right) \right] \right] \right] \right] \right) \\ \triangle \left(\left[x=0 \left[\frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f\{x\} \right] \right], \left[x=0 \left[\frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g\{x\} \right] \right] \right) \end{aligned}$$

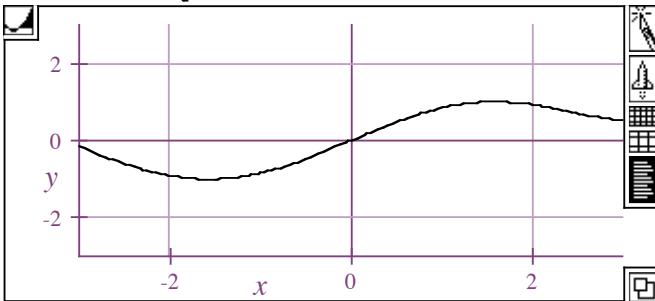
As shown above, the order of contact between $f(x)$ and the polynomial at $x = 0$ is 6. Next, the graph of the two functions.

 RC: 7/31/12: Point of contact of order 5. $g(x)$ has only 5 degrees to contribute.

8/19/12: Ooooh. I see it. Ok, I did not know to look at that. Ok. Order of contact 5.

 LS: 8/22/12: Good.

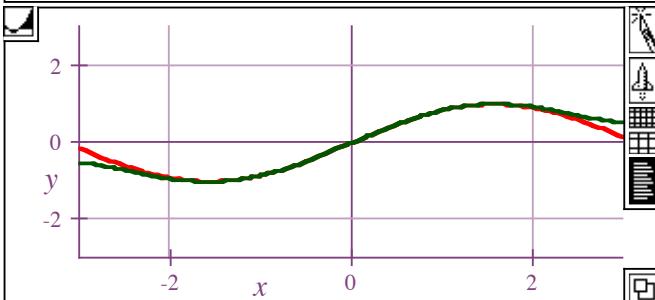
- $f(x) = \sin(x)$
 - $g(x) = -\frac{1}{6}x^3 + \frac{1}{120}x^5 + x$
 - $\text{spline}(x) = \begin{cases} f(x) & (x < 0) \\ g(x) & (x \geq 0) \end{cases}$



-3 ... 3 = left...right Stretch to Fit▼
-3 ... 3 = bottom...top cropping Moderately▼

Graph Building Blocks

Curve at $(x, \text{spline}[x])$ where $x = \text{left} \dots \text{right}$
 with a **normal** line, colored **Black**.



$-3 \dots 3 = \text{left...right}$ Stretch to Fit ▾
 $-3 \dots 3 = \text{bottom...top}$ cropping Moderately ▾

Graph Building Blocks

Curve at $(x, f[x])$ where $x = \text{left} \dots \text{right}$ with a heavy ▾ line, colored Red ▾.

Curve at $(x, g[x])$ where $x = \text{left} \dots \text{right}$ with a heavy ▾ line, colored Dark Green ▾.



G.2.b.i) Splining with $\cos(x)$

Find the polynomial of degree 2 that has the highest possible order of contact with $f(x) = \cos(x)$ at $x = 0$.

Plot the spline knotted at $\{0, 1\}$ with $f(x)$ on the right and your polynomial on the left.

Also plot on the same axes $f(x)$, the polynomial, and the spline for $-2 \leq x \leq 2$.

Describe what you see.



$f(x) = \cos(x)$

$g(x) = \sum_{k=0}^2 a_k x^k$

$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 \quad \text{Expand}$

$f(0) = g(0)$

$\triangle \cos(0) = a_0 \cdot 0^0 + a_1 \cdot 0^1 + a_2 \cdot 0^2 \quad \text{Substitute}$

$\triangle 1 = a_0 \quad \text{Simplify}$



$f(x) = \cos(x)$

$g(x) = \sum_{k=0}^2 a_k x^k$

$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 \quad \text{Expand}$



$f(x) = \cos(x)$

$g(x) = \sum_{k=0}^2 a_k x^k$

$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 \quad \text{Expand}$

$f'(0) = g'(0)$

$\triangle_{x=0} \left[\frac{d}{dx} f(x) \right] = \left[\frac{d}{dx} g(x) \right] \quad \text{Simplify}$

$$\triangle_{x=0} \left[\frac{d}{dx} \cos(x) = \right]_{x=0} \left[\frac{d}{dx} (a_0 x^0 + a_1 x^1 + a_2 x^2) \right] \quad \text{Substitute}$$

$$\triangle_{x=0} -\sin(0) = 2a_2 \cdot 0 + a_1 \quad \text{Simplify}$$

$$\triangle 0 = a_1 \quad \text{Simplify}$$

$$\square f''(0) = g''(0)$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} f[x] \right) = \right]_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} g[x] \right) \right] \quad \text{Simplify}$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \cos[x] \right) = \right]_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} [a_0 x^0 + a_1 x^1 + a_2 x^2] \right) \right] \quad \text{Substitute}$$

$$\triangle_{x=0} -\cos(0) = 2a_2 \quad \text{Simplify}$$

$$\triangle_{x=0} -1 = 2a_2 \quad \text{Simplify}$$

$$\square \quad \square a_0 = 1$$

$$\square a_1 = 0$$

$$\square 2a_2 = -1$$

$$\triangle a_2 = \frac{1}{2}(-1) \quad \text{Isolate}$$

$$\triangle a_2 = -\frac{1}{2} \quad \text{Simplify}$$

$$\square g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2$$

$$\triangle g(x) = 1x^0 + 0x^1 + \left(-\frac{1}{2}\right)x^2 \quad \text{Substitute}$$

$$\triangle g(x) = -\frac{1}{2}x^2 + 1 \quad \text{Simplify}$$



$$\square f(x) = \cos(x)$$

$$\square g(x) = -\frac{1}{2}x^2 + 1$$

$$\square (f[0], g[0])$$

$$\triangle (f[0], g[0]) = (\cos[0], -\frac{1}{2} \cdot 0^2 + 1) \quad \text{Substitute}$$

$$\triangle (f[0], g[0]) = (1, 1) \quad \text{Simplify}$$

$$\square (f'[0], g'[0])$$

$$\triangle (f'[0], g'[0]) = \left(\left[\frac{d}{dx} f[x] \right]_{x=0}, \left[\frac{d}{dx} g[x] \right]_{x=0} \right) \quad \text{Simplify}$$

$$\triangle (f'[0], g'[0]) = \left(\left[\frac{d}{dx} \cos[x] \right]_{x=0}, \left[\frac{d}{dx} \left\{ -\frac{1}{2}x^2 + 1 \right\} \right]_{x=0} \right) \quad \text{Substitute}$$

$$\triangle \left(\left[\frac{d}{dx} f[x] \right]_{x=0}, \left[\frac{d}{dx} g[x] \right]_{x=0} \right) = (-\sin[0], -0) \quad \text{Simplify}$$

$$\triangle \left(\left[\frac{d}{dx} f[x] \right]_{x=0}, \left[\frac{d}{dx} g[x] \right]_{x=0} \right) = (0, 0) \quad \text{Simplify}$$

$$\square (f''[0], g''[0])$$

$$\triangle (f''[0], g''[0]) = \left(\left[\frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \right]_{x=0}, \left[\frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \right]_{x=0} \right) \quad \text{Simplify}$$

$$\triangle (f''[0], g''[0]) = \left(\left[\frac{d}{dx} \left\{ \frac{d}{dx} \cos(x) \right\} \right]_{x=0}, \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(-\frac{1}{2}x^2 + 1 \right) \right\} \right]_{x=0} \right)$$

$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \end{array} \right] \right) = (-\cos[0], -1) \quad Simplify$$

$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \frac{d}{dx} f(x) \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \frac{d}{dx} g(x) \end{array} \right] \right) = (-1, -1) \quad Simplify$$

$(f''''[0], g''''[0])$

$$\triangle (f''''[0], g''''[0]) = \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right\} \end{array} \right] \right)$$

$$\triangle (f''''[0], g''''[0]) = \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \cos(x) \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[-\frac{1}{2}x \right] \right) \right\} \end{array} \right] \right)$$

$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right\} \end{array} \right] \right) = (\sin[0], 0)$$

$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f(x) \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g(x) \end{array} \right] \right) = (0, 0) \quad Simplify$$

$(f''''[0], g''''[0])$

$$\triangle (f''''[0], g''''[0]) = \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right) \right\} \end{array} \right] \right)$$

$$\triangle (f''''[0], g''''[0]) = \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \cos(x) \right) \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \right) \right) \right\} \end{array} \right] \right)$$

$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right\} \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right) \right\} \end{array} \right] \right) =$$

$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f(x) \end{array} \right], \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g(x) \end{array} \right] \right) = (1, 0) \quad Si$$

As shown above, the order of contact between $f(x)$ and the polynomial at $x = 0$ is 3. Next, the graph of the two functions.

RC: 7/31/12: Point of contact of order 2. $g(x)$ has only 2 degrees to contribute.

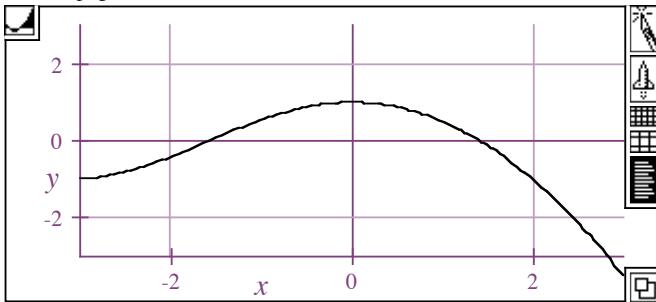
8/19/12: I see it now. Only an order of contact of 2.

LS: 8/22/12: Good.

$\boxed{\bullet} f(x) = \cos(x)$

$\boxed{\bullet} g(x) = -\frac{1}{2}x^2 + 1$

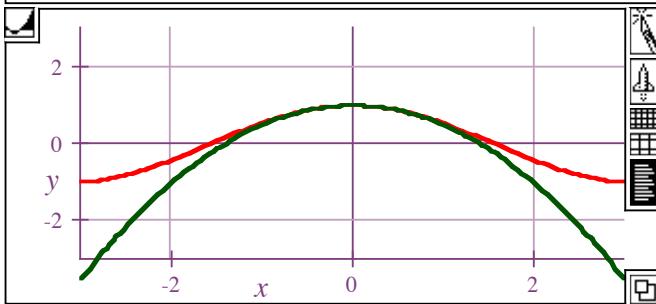
$\boxed{\bullet} \text{spline}(x) = \begin{cases} f(x) & (x < 0) \\ g(x) & (x \geq 0) \end{cases}$



$-3 \dots 3 = \text{left...right}$ Stretch to Fit ▾
 $-3 \dots 3 = \text{bottom...top}$ cropping Moderately ▾

Graph Building Blocks

Curve at $(x, \text{spline}[x])$ where $x = \text{left} \dots \text{right}$
 with a normal ▾ line, colored Black ▾.



$-3 \dots 3 = \text{left...right}$ Stretch to Fit ▾
 $-3 \dots 3 = \text{bottom...top}$ cropping Moderately ▾

Graph Building Blocks

Curve at $(x, f[x])$ where $x = \text{left} \dots \text{right}$ with a
 heavy ▾ line, colored Red ▾.

Curve at $(x, g[x])$ where $x = \text{left} \dots \text{right}$ with a
 heavy ▾ line, colored Dark Green ▾.

G.2.b.ii)

Find the polynomial of degree 4 that has the highest possible order of contact with $f(x) = \cos(x)$ at $x = 0$.

Plot the spline knotted at $\{0,1\}$ with $f(x)$ on the right and your polynomial on the left.
 Also plot on the same axes $f(x)$, the polynomial, and the spline for $-2 \leq x \leq 2$.

Describe what you see, paying special attention to a comparison of the plots in parts



$f(\pi) = \cos(\pi)$

$$\square g(x) = \sum_{k=0}^4 a_k x^k$$

$$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 \quad \text{Expand}$$

$$\triangle g(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 \quad \text{Simplify}$$

$$\square f(0) = g(0)$$

$$\triangle \cos(0) = a_4 \cdot 0^4 + a_3 \cdot 0^3 + a_2 \cdot 0^2 + a_1 \cdot 0 + a_0 \quad \text{Substitute}$$

$$\triangle 1 = a_0 \quad \text{Simplify}$$

$$\square f(x) = \cos(x)$$

$$\square g(x) = \sum_{k=0}^4 a_k x^k$$

$$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4$$

$$\triangle g(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$\square f'(0) = g'(0)$$

$$\triangle \underset{x=0}{\left[\frac{d}{dx} f(x) \right]} = \underset{x=0}{\left[\frac{d}{dx} g(x) \right]} \quad \text{Simplify}$$

$$\triangle \underset{x=0}{\left[\frac{d}{dx} \cos(x) \right]} = \underset{x=0}{\left[\frac{d}{dx} (a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0) \right]} \quad \text{Substitut}$$

$$\triangle -\sin(0) = 4a_4 \cdot 0^3 + 3a_3 \cdot 0^2 + 2a_2 \cdot 0 + a_1 \quad \text{Simplify}$$

$$\triangle 0 = a_1 \quad \text{Simplify}$$

$$\square f''(0) = g''(0)$$

$$\triangle \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right]} = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right]} \quad \text{Simplify}$$

$$\triangle \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \cos(x) \right) \right]} = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} (a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0) \right) \right]}$$

$$\triangle -\cos(0) = 12a_4 \cdot 0^2 + 6a_3 \cdot 0 + 2a_2 \quad \text{Simplify}$$

$$\triangle -1 = 2a_2 \quad \text{Simplify}$$

$$\square f'''(0) = g'''(0)$$

$$\triangle \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right) \right]} = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right) \right]} \quad \text{Simplify}$$

$$\triangle \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \cos(x) \right) \right) \right]} = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} (a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0) \right) \right) \right]}$$

$$\triangle \sin(0) = 24a_4 \cdot 0 + 6a_3 \quad \text{Simplify}$$

$$\triangle 0 = 6a_3 \quad \text{Simplify}$$

$$\square f''''(0) = g''''(0)$$

$$\triangle \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right) \right) \right]} = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right) \right) \right]} \quad \text{Simplify}$$

$$\triangle \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \cos(x) \right) \right) \right) \right]} = \underset{x=0}{\left[\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} (a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0) \right) \right) \right) \right]}$$

$$\triangle \cos(0) = 24 a_4 \quad \text{Simplify}$$

$$\triangle 1 = 24 a_4 \quad \text{Simplify}$$

$\square a_0 = 1$

$\square a_1 = 0$

$\square 2a_2 = -1$

$$\triangle a_2 = \frac{1}{2}(-1) \quad \text{Isolate}$$

$$\triangle a_2 = -\frac{1}{2} \quad \text{Simplify}$$

$\square 6a_3 = 0$

$$\triangle a_3 = \frac{1}{6} \cdot 0 \quad \text{Isolate}$$

$$\triangle a_3 = 0 \quad \text{Simplify}$$

$\square 24a_4 = 1$

$$\triangle a_4 = \frac{1}{24} \cdot 1 \quad \text{Isolate}$$

$$\triangle a_4 = \frac{1}{24} \quad \text{Simplify}$$

$\square g(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$

$$\triangle g(x) = \frac{1}{24} x^4 + 0 x^3 + \left(-\frac{1}{2}\right) x^2 + 0 x + 1 \quad \text{Substitute}$$

$$\triangle g(x) = -\frac{1}{2} x^2 + \frac{1}{24} x^4 + 1 \quad \text{Simplify}$$

$f(x) = \cos(x)$

$g(x) = -\frac{1}{2} x^2 + \frac{1}{24} x^4 + 1$

$(f[0], g[0])$

$$\triangle (f[0], g[0]) = \left(\cos[0], -\frac{1}{2} \cdot 0^2 + \frac{1}{24} \cdot 0^4 + 1\right) \quad \text{Substitute}$$

$$\triangle (f[0], g[0]) = (1, 1) \quad \text{Simplify}$$

$(f'[0], g'[0])$

$$\triangle (f'[0], g'[0]) = \left(\left[x=0 \left[\frac{d}{dx} f(x) \right] \right], \left[x=0 \left[\frac{d}{dx} g(x) \right] \right] \right) \quad \text{Simplify}$$

$$\triangle (f'[0], g'[0]) = \left(\left[x=0 \left[\frac{d}{dx} \cos(x) \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ -\frac{1}{2} x^2 + \frac{1}{24} x^4 + 1 \right\} \right] \right] \right) \quad \text{Substi}$$

$$\triangle \left(\left[x=0 \left[\frac{d}{dx} f(x) \right] \right], \left[x=0 \left[\frac{d}{dx} g(x) \right] \right] \right) = \left(-\sin[0], \frac{1}{6} \cdot 0^3 - 0 \right) \quad \text{Simplify}$$

$$\triangle \left(\left[x=0 \left[\frac{d}{dx} f(x) \right] \right], \left[x=0 \left[\frac{d}{dx} g(x) \right] \right] \right) = (0, 0) \quad \text{Simplify}$$

$(f''[0], g''[0])$

$$\triangle (f''[0], g''[0]) = \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \right] \right] \right) \quad \text{Simplify}$$

$$\triangle (f''[0], g''[0]) = \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \cos(x) \right\} \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(-\frac{1}{2} x^2 + \frac{1}{24} x^4 + 1 \right) \right\} \right] \right] \right)$$

$$\triangle \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \right] \right] \right) = \left(-\cos[0], \frac{1}{2} \cdot 0^2 - 1 \right) \quad \text{Simpli}$$

$$\square(f'''[0], g'''[0]) \quad \text{Simplify}$$

$$\triangle(f'''[0], g'''[0]) = \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} g[x] \right) \right\} \right] \right)$$

$$\triangle(f'''[0], g'''[0]) = \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \cos[x] \right) \right\} \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[-\frac{1}{2} x^2 \right] \right) \right\} \right] \right]$$

$$\triangle \left[\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right|_{x=0}, \left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} g[x] \right) \right\} \right|_{x=0} \right] = (\sin[0], 0)$$

$$\triangleleft_{x=0} \left(\frac{d}{dx} x \frac{d}{dx} \frac{d}{dx} f\{x\}, \left. \frac{d}{dx} x \frac{d}{dx} \frac{d}{dx} g\{x\} \right|_{x=0} \right) = (0,0) \quad Simplify$$

$$\square(f''''[0], g'''[0])$$

$$\triangle(f''''[0], g''''[0]) = \left[\left. x = 0 \right| \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f\{x\} \right] \right) \right\} \right], \left[\left. x = 0 \right| \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g\{x\} \right] \right) \right\} \right]$$

$$\triangle(f''''[0], g''''[0]) = \left(\left. x = 0 \right| \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \cos \{x\} \right) \right) \right\}, \left. x = 0 \right| \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \cos \{x\} \right) \right) \right\} \right)$$

$$\triangle \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f\{x\} \right] \right\} \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g\{x\} \right] \right\} \right] \right] \right) =$$

$$\triangle \left[x=0 \left| \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f\{x\} \right. \right], \left. x=0 \left| \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g\{x\} \right. \right] = (1,1) \quad \text{Si}$$

$$\square(f''''[0], g''''[0])$$

$$\triangle(f''''[0], g''''[0]) = \left(\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right\} \right\} \right\} \right|_{x=0}, \left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right) \right\} \right\} \right|_{x=0} \right)$$

$$\triangle(f''''[0], g''''[0]) = \left(\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left\{ \frac{d}{dx} \cos(x) \right\} \right) \right\} \right|_{x=0}, \left. \frac{d}{dx} \right|_{x=0} \right)$$

$$\triangle \left[\left. x = 0 \right| \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \right] \right\} \right], \left. x = 0 \right| \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \right] \right\} \right].$$

$$\triangle \left(\left[\begin{array}{c} x=0 \\ \frac{d}{dx} \end{array} \right] \frac{d}{dx} \left[\begin{array}{c} d \\ d \\ d \\ d \\ d \end{array} \right] f\{x\} \right), \left[\begin{array}{c} x=0 \\ \frac{d}{dx} \end{array} \right] \frac{d}{dx} \left[\begin{array}{c} d \\ d \\ d \\ d \\ d \end{array} \right] g\{x\} \right) = 0$$

$$\square(f'''''[0], g'''''[0])$$

$$\triangle(f''''''[0], g''''''[0]) = \left(\left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right] \right\} \right] \right) \Big|_{x=0}, \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} g[x] \right) \right\} \right] \right\} \right] \right) \Big|_{x=0} \right)$$

$$\triangle(f''''''[0], g''''''[0]) = \left[\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \cos[x] \right) \right\} \right) \right\} \right\} \right]_{x=0},$$

$$\triangleleft \left(\left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right\} \right\} \right] \right], \left[x=0 \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right\} \right] \right] \right] \right]$$

$$\triangle \int \left[\frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f\{x\} \right], \quad \left[\frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g\{x\} \right]$$

 As shown above, the order of contact between $f(x)$

and the polynomial at $x = 0$ is 5. Next, the graph of the two functions.

 RC: 7/31/12: You fix remaining issues as described above.

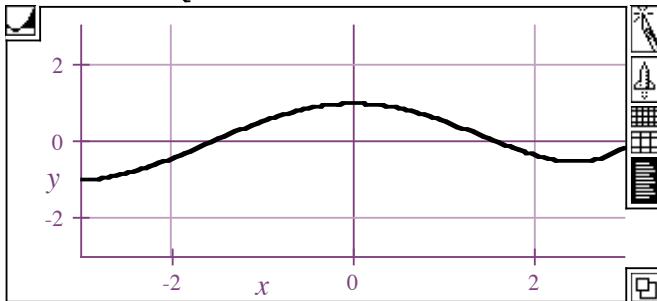
8/19/12: Ok, so $g(x)$ can only contribute 4 orders of contact. I understand. Max for this set: 4.

LS: 8/22/12: Good.

$f(x) = \cos(x)$

$g(x) = -\frac{1}{2}x^2 + \frac{1}{24}x^4 + 1$

$\text{spline}(x) = \begin{cases} f(x) & (x < 0) \\ g(x) & (x \geq 0) \end{cases}$

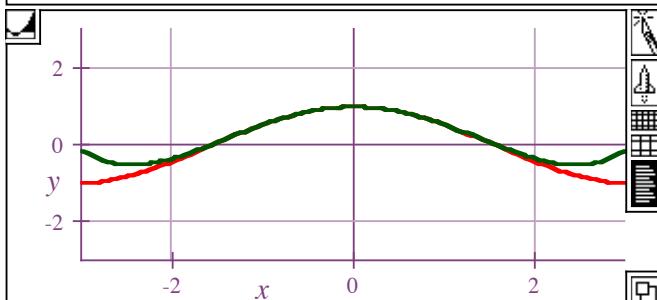


-3 ... 3 = left...right Stretch to Fit▼

-3 ... 3 = bottom...top cropping Moderately▼

Graph Building Blocks

Curve at $(x, \text{spline}[x])$ where $x = \text{left} \dots \text{right}$
with a heavy▼ line, colored Black▼.



-3 ... 3 = left...right Stretch to Fit▼

-3 ... 3 = bottom...top cropping Moderately▼

Graph Building Blocks

Curve at $(x, f[x])$ where $x = \text{left} \dots \text{right}$ with a
heavy▼ line, colored Red▼.

Curve at $(x, g[x])$ where $x = \text{left} \dots \text{right}$ with a
heavy▼ line, colored Dark Green▼.



G.2.c.i) Splining with $\frac{1}{1-x}$

Find the polynomial of degree 2 that has the highest possible order of contact with

$$f(x) = \frac{1}{1-x} \text{ at } x = 0.$$

Plot the spline knotted at $\{0,1\}$ with $f(x)$ on the right and your polynomial on the left

Also plot on the same axes $f(x)$, the polynomial, and the spline for $-0.9 \leq x \leq 0.9$.

Describe what you see.



$$\square f(x) = \frac{1}{1-x}$$

$$\square g(x) = \sum_{k=0}^2 a_k x^k$$

$$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 \quad \text{Expand}$$

$$\triangle g(x) = a_2 x^2 + a_1 x + a_0 \quad \text{Simplify}$$

$$\square f(0) = g(0)$$

$$\triangle \frac{1}{1-0} = a_2 \cdot 0^2 + a_1 \cdot 0 + a_0 \quad \text{Substitute}$$

$$\triangle 1 = a_0 \quad \text{Simplify}$$



$$\square f(x) = \frac{1}{1-x}$$

$$\square g(x) = \sum_{k=0}^2 a_k x^k$$

$$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2$$

$$\triangle g(x) = a_2 x^2 + a_1 x + a_0$$

$$\square f'(0) = g'(0)$$

$$\triangle \left. \frac{d}{dx} f(x) \right|_{x=0} = \left. \frac{d}{dx} g(x) \right|_{x=0} \quad \text{Simplify}$$

$$\triangle \left. \frac{d}{dx} \frac{1}{1-x} \right|_{x=0} = \left. \frac{d}{dx} (a_2 x^2 + a_1 x + a_0) \right|_{x=0} \quad \text{Substitute}$$

$$\triangle \frac{1}{(-0+1)^2} = 2a_2 \cdot 0 + a_1 \quad \text{Simplify}$$

$$\triangle 1 = a_1 \quad \text{Simplify}$$

$$\square f''(0) = g''(0)$$

$$\triangle \left. \frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right|_{x=0} = \left. \frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right|_{x=0} \quad \text{Simplify}$$

$$\triangle \left. \frac{d}{dx} \left(\frac{d}{dx} \frac{1}{1-x} \right) \right|_{x=0} = \left. \frac{d}{dx} \left(\frac{d}{dx} (a_2 x^2 + a_1 x + a_0) \right) \right|_{x=0} \quad \text{Substitute}$$

$$\triangle 2 \frac{1}{(-0+1)^3} = 2a_2 \quad \text{Simplify}$$

$$\triangle 2 = 2 a_2 \quad Simplify$$

$a_0 = 1$

$a_1 = 1$

$2 a_2 = 2$

$$\triangle a_2 = \frac{1}{2} \cdot 2 \quad Isolate$$

$$\triangle a_2 = 1 \quad Simplify$$

$g(x) = a_2 x^2 + a_1 x + a_0$

$$\triangle g(x) = 1 x^2 + 1 x + 1 \quad Substitute$$

$$\triangle g(x) = x^2 + x + 1 \quad Simplify$$

$f(x) = \frac{1}{1-x}$

$g(x) = x^2 + x + 1$

$(f[0], g[0])$

$$\triangle (f[0], g[0]) = \left(\frac{1}{1-0}, 0^2 + 0 + 1 \right) \quad Substitute$$

$$\triangle (f[0], g[0]) = (1, 1) \quad Simplify$$

$(f'[0], g'[0])$

$$\triangle (f'[0], g'[0]) = \left(\left[\begin{array}{l} x=0 \\ \frac{d}{dx} f(x) \end{array} \right], \left[\begin{array}{l} x=0 \\ \frac{d}{dx} g(x) \end{array} \right] \right) \quad Simplify$$

$$\triangle (f'[0], g'[0]) = \left(\left[\begin{array}{l} x=0 \\ \frac{d}{dx} \frac{1}{1-x} \end{array} \right], \left[\begin{array}{l} x=0 \\ \frac{d}{dx} (x^2 + x + 1) \end{array} \right] \right) \quad Substitute$$

$$\triangle \left(\left[\begin{array}{l} x=0 \\ \frac{d}{dx} f(x) \end{array} \right], \left[\begin{array}{l} x=0 \\ \frac{d}{dx} g(x) \end{array} \right] \right) = \left(\frac{1}{[-0+1]^2}, 2 \cdot 0 + 1 \right) \quad Simplify$$

$$\triangle \left(\left[\begin{array}{l} x=0 \\ \frac{d}{dx} f(x) \end{array} \right], \left[\begin{array}{l} x=0 \\ \frac{d}{dx} g(x) \end{array} \right] \right) = (1, 1) \quad Simplify$$

$(f''[0], g''[0])$

$$\triangle (f''[0], g''[0]) = \left(\left[\begin{array}{l} x=0 \\ \frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \end{array} \right], \left[\begin{array}{l} x=0 \\ \frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \end{array} \right] \right) \quad Simplify$$

$$\triangle (f''[0], g''[0]) = \left(\left[\begin{array}{l} x=0 \\ \frac{d}{dx} \left(\frac{d}{dx} \frac{1}{1-x} \right) \end{array} \right], \left[\begin{array}{l} x=0 \\ \frac{d}{dx} \left(\frac{d}{dx} (x^2 + x + 1) \right) \end{array} \right] \right)$$

$$\triangle \left(\left[\begin{array}{l} x=0 \\ \frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \end{array} \right], \left[\begin{array}{l} x=0 \\ \frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \end{array} \right] \right) = \left(2 \frac{1}{[-0+1]^3}, 2 \right) \quad Simplify$$

$$\triangle \left(\left[\begin{array}{l} x=0 \\ \frac{d}{dx} \frac{d}{dx} f(x) \end{array} \right], \left[\begin{array}{l} x=0 \\ \frac{d}{dx} \frac{d}{dx} g(x) \end{array} \right] \right) = (2, 2) \quad Simplify$$

$(f'''[0], g'''[0])$

$$\triangle (f'''[0], g'''[0]) = \left(\left[\begin{array}{l} x=0 \\ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} f(x) \right) \right) \end{array} \right], \left[\begin{array}{l} x=0 \\ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} g(x) \right) \right) \end{array} \right] \right)$$

$$\triangle (f'''[0], g'''[0]) = \left(\left[\begin{array}{l} x=0 \\ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} \frac{1}{1-x} \right) \right) \end{array} \right], \left[\begin{array}{l} x=0 \\ \frac{d}{dx} \left(\frac{d}{dx} \left(\frac{d}{dx} (x^2 + x + 1) \right) \right) \end{array} \right] \right)$$

$$\boxed{\left[\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right\} \right]_{x=0}, \left[\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} g[x] \right) \right\} \right]_{x=0}} = \left(6, \frac{1}{[-0+1]^4} \right)$$

$$\boxed{\left[\left. \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f[x] \right]_{x=0}, \left[\left. \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g[x] \right]_{x=0} \right) = (6,0)}$$

As shown above, the order of contact between $f(x)$ and the polynomial at $x = 0$ is 2. Next, the graph of the two functions.

RC: 7/31/12: Good

$f(x) = \frac{1}{1-x}$

$g(x) = x^2 + x + 1$

spline(x) = $\begin{cases} f(x) & (x < 0) \\ g(x) & (x \geq 0) \end{cases}$

-3 ... 3 = left...right Stretch to Fit▼
-3 ... 3 = bottom...top cropping Moderately▼

Graph Building Blocks

Curve at $(x, \text{spline}[x])$ where $x = \text{left} \dots \text{right}$ with a heavy▼ line, colored Black▼.

-3 ... 3 = left...right Stretch to Fit▼
-3 ... 3 = bottom...top cropping Moderately▼

Graph Building Blocks

Curve at $(x, f[x])$ where $x = \text{left} \dots \text{right}$ with a heavy▼ line, colored Red▼.

↳ Curve at $(x, g[x])$ where $x = \text{left ... right}$ with a
heavy line, colored Dark Green.



G.2.c.ii)

Find the polynomial of degree 4 that has the highest possible order of contact with

$$f(x) = \frac{1}{1-x} \text{ at } x = 0.$$

Plot the spline knotted at $\{0,1\}$ with $f(x)$ on the right and your polynomial on the left

Also plot on the same axes $f(x)$, the polynomial, and the spline for $-0.9 \leq x \leq 0.9$.

Describe what you see, paying special attention to a comparison of the plots in parts



$$\square f(x) = \frac{1}{1-x}$$

$$\square g(x) = \sum_{k=0}^4 a_k x^k$$

$$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 \quad \text{Expand}$$

$$\triangle g(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 \quad \text{Simplify}$$

$$\square f(0) = g(0)$$

$$\triangle \frac{1}{1-0} = a_4 \cdot 0^4 + a_3 \cdot 0^3 + a_2 \cdot 0^2 + a_1 \cdot 0 + a_0 \quad \text{Substitute}$$

$$\triangle 1 = a_0 \quad \text{Simplify}$$



$$\square f(x) = \frac{1}{1-x}$$

$$\square g(x) = \sum_{k=0}^4 a_k x^k$$

$$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 \quad \text{Expand}$$

$$\triangle g(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 \quad \text{Simplify}$$

$$\square f'(0) = g'(0)$$

$$\triangle \left. \frac{d}{dx} f(x) \right|_{x=0} = \left. \frac{d}{dx} g(x) \right|_{x=0} \quad \text{Simplify}$$

$$\triangle \left. \frac{d}{dx} \frac{1}{1-x} \right|_{x=0} = \left. \frac{d}{dx} (a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0) \right|_{x=0} \quad \text{Substitute}$$

$$\triangle \frac{1}{(-0+1)^2} = 4 a_4 \cdot 0^3 + 3 a_3 \cdot 0^2 + 2 a_2 \cdot 0 + a_1 \quad \text{Simplify}$$

$$\triangle 1 = a_1 \quad \text{Simplify}$$

$$\square f''(0) = g''(0)$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right] = \triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} g[x] \right) \right] \quad Simplify$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \frac{1}{1-x} \right) \right] = \triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} [a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0] \right) \right]$$

$$\triangle 2 \frac{1}{(-0+1)^3} = 12 a_4 \cdot 0^2 + 6 a_3 \cdot 0 + 2 a_2 \quad Simplify$$

$$\triangle 2 = 2 a_2 \quad Simplify$$

$$\square f'''(0) = g'''(0)$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f[x] \right] \right) \right] = \triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g[x] \right] \right) \right] \quad Simplify$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \frac{1}{1-x} \right] \right) \right] = \triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \{a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0\} \right] \right) \right]$$

$$\triangle 6 \frac{1}{(-0+1)^4} = 24 a_4 \cdot 0 + 6 a_3 \quad Simplify$$

$$\triangle 6 = 6 a_3 \quad Simplify$$

$$\square f''''(0) = g''''(0)$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left[\frac{d}{dx} f(x) \right] \right] \right) \right] = \triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left[\frac{d}{dx} g(x) \right] \right] \right) \right] \quad Simplify$$

$$\triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left[\frac{d}{dx} \frac{1}{1-x} \right] \right] \right) \right] = \triangle_{x=0} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left[\frac{d}{dx} (a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0) \right] \right] \right) \right]$$

$$\triangle 24 \frac{1}{(-0+1)^5} = 24 a_4 \quad Simplify$$

$$\triangle 24 = 24 a_4 \quad Simplify$$

$a_0 = 1$

$a_1 = 1$

$2 a_2 = 2$

$$\triangle a_2 = \frac{1}{2} \cdot 2$$

$$\triangle a_2 = 1$$

$6 a_3 = 6$

$$\triangle a_3 = \frac{1}{6} \cdot 6 \quad Isolate$$

$$\triangle a_3 = 1 \quad Simplify$$

$24 a_4 = 24$

$$\triangle a_4 = \frac{1}{24} \cdot 24 \quad Isolate$$

$$\triangle a_4 = 1 \quad Simplify$$

$g(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$

$$\triangle g(x) = \left(\frac{1}{24} \cdot 24\right) x^4 + \left(\frac{1}{6} \cdot 6\right) x^3 + 1 x^2 + 1 x + 1 \quad Substitute$$

$$\triangle g(x) = x^4 + x^3 + x^2 + x + 1 \quad Simplify$$



$f(x) = \frac{1}{1-x}$

$$\square g(x) = x^4 + x^3 + x^2 + x + 1$$

$$\square (f[0], g[0])$$

$$\triangle (f[0], g[0]) = \left(\frac{1}{1-0}, 0^4 + 0^3 + 0^2 + 0 + 1 \right) \quad \text{Substitute}$$

$$\triangle (f[0], g[0]) = (1, 1) \quad \text{Simplify}$$

$$\square (f'[0], g'[0])$$

$$\triangle (f'[0], g'[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} f[x] \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} g[x] \\ x=0 \end{array} \right] \right) \quad \text{Simplify}$$

$$\triangle (f'[0], g'[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \frac{1}{1-x} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \{x^4 + x^3 + x^2 + x + 1\} \\ x=0 \end{array} \right] \right) \quad \text{Sub}$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} f[x] \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} g[x] \\ x=0 \end{array} \right] \right) = \left(\frac{1}{[-0+1]^2}, 4 \cdot 0^3 + 3 \cdot 0^2 + 2 \cdot 0 + 1 \right)$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} f[x] \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} g[x] \\ x=0 \end{array} \right] \right) = (1, 1) \quad \text{Simplify}$$

$$\square (f''[0], g''[0])$$

$$\triangle (f''[0], g''[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \\ x=0 \end{array} \right] \right) \quad \text{Simplify}$$

$$\triangle (f''[0], g''[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \frac{1}{1-x} \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} (x^4 + x^3 + x^2 + x) \right\} \\ x=0 \end{array} \right] \right)$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} f(x) \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} g(x) \right\} \\ x=0 \end{array} \right] \right) = \left(2 \frac{1}{[-0+1]^3}, 12 \cdot 0^2 + 6 \cdot 0 \right)$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} \frac{d}{dx} f[x] \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \frac{d}{dx} g[x] \\ x=0 \end{array} \right] \right) = (2, 2) \quad \text{Simplify}$$

$$\square (f'''[0], g'''[0])$$

$$\triangle (f'''[0], g'''[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} f[x] \right\} \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} g[x] \right\} \right\} \\ x=0 \end{array} \right] \right)$$

$$\triangle (f'''[0], g'''[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \frac{1}{1-x} \right\} \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} (x^4 + x^3 + x^2 + x) \right\} \right\} \\ x=0 \end{array} \right] \right)$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} f[x] \right\} \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} g[x] \right\} \right\} \\ x=0 \end{array} \right] \right) = \left(6 \frac{1}{[-0+1]^4} \right)$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f[x] \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g[x] \\ x=0 \end{array} \right] \right) = (6, 6) \quad \text{Simplify}$$

$$\square (f''''[0], g''''[0])$$

$$\triangle (f''''[0], g''''[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} f[x] \right\} \right\} \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} g[x] \right\} \right\} \right\} \\ x=0 \end{array} \right] \right)$$

$$\triangle (f''''[0], g''''[0]) = \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \frac{1}{1-x} \right\} \right\} \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} (x^4 + x^3 + x^2 + x) \right\} \right\} \right\} \\ x=0 \end{array} \right] \right)$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} f[x] \right\} \right\} \right\} \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} \left\{ \frac{d}{dx} g[x] \right\} \right\} \right\} \\ x=0 \end{array} \right] \right) =$$

$$\triangle \left(\left[\begin{array}{c} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f[x] \\ x=0 \end{array} \right], \left[\begin{array}{c} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g[x] \\ x=0 \end{array} \right] \right) = (24, 24)$$

$$\square (f''''[0], g''''[0])$$

$$\triangle(f''''[0], g''''[0]) = \left[\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f(x) \right] \right) \right\} \right|_{x=0}, \left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g(x) \right] \right) \right\} \right|_{x=0} \right]$$

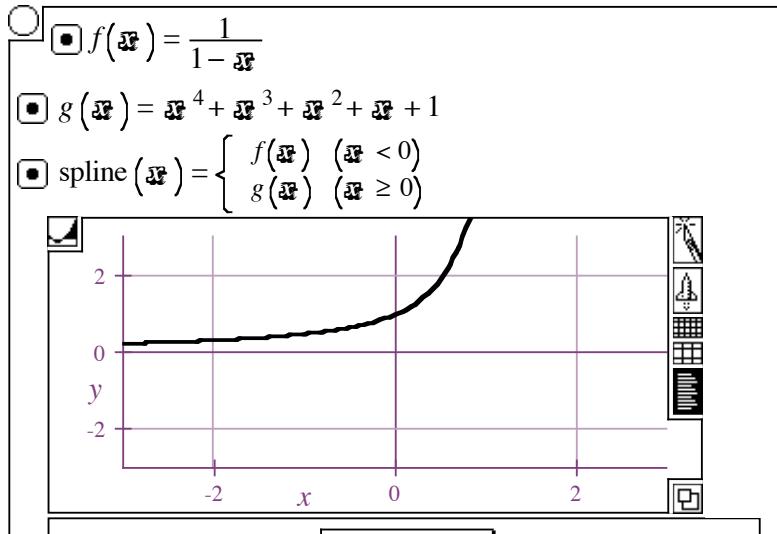
$$\triangle(f''''[0], g''''[0]) = \left[\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \frac{1}{1-x} \right] \right) \right\} \right|_{x=0}, \left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \frac{1}{1-x} \right] \right) \right\} \right|_{x=0} \right]$$

$$\triangle \left[\left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f(x) \right] \right) \right\} \right|_{x=0}, \left. \frac{d}{dx} \left\{ \frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g(x) \right] \right) \right\} \right|_{x=0} \right]$$

$$\triangle \left[\left. \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} f(x) \right|_{x=0}, \left. \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} g(x) \right|_{x=0} \right] = (1)$$

As shown above, the order of contact between $f(x)$ and the polynomial at $x = 0$ is 4. Next, the graph of the two functions.

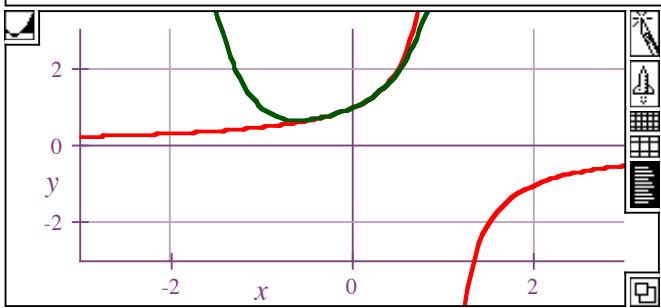
RC: 7/31/12: Good



-3 ... 3 = left...right Stretch to Fit▼
-3 ... 3 = bottom...top cropping Moderately▼

Graph Building Blocks

Curve at (x , spline [x]) where $x = \text{left} \dots \text{right}$
with a heavy▼ line, colored Black▼.



-3 ... 3 = left...right Stretch to Fit▼
-3 ... 3 = bottom...top cropping Moderately▼

Graph Building Blocks

- ↳ Curve at $(x, f[x])$ where $x = \text{left} \dots \text{right}$ with a **heavy** line, colored **Red**.
- ↳ Curve at $(x, g[x])$ where $x = \text{left} \dots \text{right}$ with a **heavy** line, colored **Dark Green**.



G.2.d.i) Splining with $\text{Erf}(x-1)$

Find the polynomial of degree 1 that has the highest possible order of contact with $f(x) = \text{Erf}(x - 1)$ at $x = 1$.

Plot the spline knotted at $\{1, 0\}$ with $f(x)$ on the right and your polynomial on the left. Also plot on the same axes $f(x)$, the polynomial, and the spline for $0 \leq x \leq 2$. Describe what you see.

Hint

Remember that $\text{Erf}(x)$ is defined as:

$$\text{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-(t^2)} dt$$

07/12/12: Question: Why is that the integral will not work out? This may be a silly question, but I seem to be blanking on integration (temporarily I swear) and cannot seem to get the integral to work. Would love to be able to finish this.

RC: 7/31/12: Some integrals just do not have algebraic integrals (antiderivatives). That's what Calculus III is the study of: what to do when algebraic integration gets nowhere.

8/1/12: Then my question becomes what do I do? I cannot very well solve for the variables of $g(x)$ if I am unable to set them equal... I know that the function $\text{erf}(x-1)$ has a plottable graph, but I am unsure how to get $g(x)$ from it.



$\text{Erf}(x) = \frac{2}{\sqrt{\pi}} \left(\int_0^x e^{-t^2} dt \right)$

$f(x) = \text{Erf}(x - 1)$

$\Delta f(x) = \frac{2}{\sqrt{\pi}} \left(\int_0^{x-1} e^{-t^2} dt \right)$

$$\triangle f(x) = 2 \frac{\int_0^{x-1} e^{-t^2} dt}{\sqrt{\pi}}$$

$g(x) = \sum_{k=0}^1 a_k x^k$

$$\triangle g(x) = a_0 x^0 + a_1 x^1$$

$$\triangle g(x) = a_1 x + a_0$$

RC: 7/31/12: Should we be looking at x=0 or somewhere else here?

$a_1 = 0$

$f(1) = g(1)$

$$\triangle 2 \frac{\int_0^{1-1} e^{-t^2} dt}{\sqrt{\pi}} = a_1 \cdot 1 + a_0 \quad \text{Substitute}$$

$$\triangle 2 \frac{\int_0^{1-1} e^{-t^2} dt}{\sqrt{\pi}} = a_1 + a_0 \quad \text{Simplify}$$

$$\triangle 2 \frac{\int_0^{1-1} e^{-t^2} dt}{\sqrt{\pi}} = 2 \frac{1}{\sqrt{\pi}} + a_0 \quad \text{Substitute}$$

$$\triangle 2 \frac{\int_0^0 e^{-t^2} dt}{\sqrt{\pi}} = 2 \frac{1}{\sqrt{\pi}} + a_0 \quad \text{Simplify}$$

$$\square 2 \frac{0}{\sqrt{\pi}} = 2 \frac{1}{\sqrt{\pi}} + a_0$$

$$\triangle 0 = 2 \frac{1}{\sqrt{\pi}} + a_0 \quad \text{Simplify}$$

$$\triangle a_0 = 0 - 2 \frac{1}{\sqrt{\pi}} \quad \text{Isolate}$$

$$\blacksquare a_0 = -2 \frac{1}{\sqrt{\pi}} \quad \text{Simplify}$$

LS: 8/17/12: As stated before - the left side is 0 since the upper bound matches the lower bound. But, you won't be able to get a_0 solved for until you find the a_1 value. See hint below.

RC: 8/13/12: The left side will be 0 because lowerlimit = upperlimit. Then go forward and finish up the first and second derivatives. Remember:

$$f'(x) = \frac{2}{\sqrt{\pi}} e^{-(x-1)^2}$$

$$\square f'(\mathfrak{x}) = \frac{2}{\sqrt{\pi}} e^{(\mathfrak{x}-1)^1}$$

$$\square f'(1) = g'(1)$$

$$\triangle_{x=1} \left[\frac{d}{dx} f(x) \right] = \underset{x=1}{\left[\frac{d}{dx} g(x) \right]} \quad Simplify$$

$$\triangle_{x=1} \left[\frac{d}{dx} \left(2 \frac{\int_0^{x-1} e^{-t^2} dt}{\sqrt{\pi}} \right) \right] = \underset{x=1}{\left[\frac{d}{dx} (a_1 x + a_0) \right]} \quad Substitute$$

$$\triangle_{x=1} \left[2 \frac{1}{\pi^{\frac{1}{2}}} \frac{d}{dx} \left(\int_0^{x-1} e^{-t^2} dt \right) \right] = a_1 \quad Simplify$$

$$\triangle_{x=1} \left[2 \frac{1}{\sqrt{\pi}} \frac{d}{dx} \left(\int_0^{x-1} e^{-t^2} dt \right) \right] = a_1 \quad Simplify$$

$$\square \frac{2}{\sqrt{\pi}} e^{(1-1)^1} = a_1$$

$$\triangle 2 \frac{1}{\sqrt{\pi}} = a_1 \quad Simplify$$

$$\blacksquare a_1 = 2 \frac{1}{\sqrt{\pi}}$$

☞ LS: 8/17/12: This is not $f'(1)$ on the left, because you have not plugged in 1 for x . You will have the value of a_1 after you plug in your value to the f' function.

☞ LS: 8/22/12: When you plug in 1, $f'(1) = 2/\sqrt{\pi} * e^{(1-1)} = 2/\sqrt{\pi} * e^0 = 2/\sqrt{\pi}$. So, $a_1 = 2/\sqrt{\pi}$

Now go from there.

$$\square_{x=1} \left[\frac{d}{dx} \left(2 \frac{\int_0^{1-1} e^{-t^2} dt}{\sqrt{\pi}} \right) \right] = \underset{x=1}{\left[\frac{d}{dx} (a_1 x + a_0) \right]}$$

$$\triangle_{x=1} \left[2 \frac{1}{\pi^{\frac{1}{2}}} \frac{d}{dx} \left(\int_0^{1-1} e^{-t^2} dt \right) \right] = a_1 \quad Simplify$$

$$\square_{x=1} \left[2 \frac{1}{\pi^{\frac{1}{2}}} \frac{d}{dx} 0 \right] = a_1$$

$$\triangle 0 = a_1 \quad Simplify$$

☞ **8/19/12: So $a_1 = 0$ and $a_0 = 0$... Does that mean that $f(x) = 0$ as well? I don't appear to be understanding this.... My math is all funky.**

$$\square \frac{d}{dx} \left(\frac{2}{\sqrt{\pi}} e^{[\mathfrak{x}-1]^1} \right)$$

$$\triangle \frac{d}{dx} \left(\frac{2}{\sqrt{\pi}} e^{[\mathfrak{x}-1]^2} \right) = 2 \frac{1}{\pi^{\frac{1}{2}}} e^{[\mathfrak{x}-1]^2} \frac{d}{dx} \mathfrak{x} \quad \text{Simplify}$$

$$\triangle \frac{d}{dx} \left(\frac{2}{\sqrt{\pi}} e^{[\mathfrak{x}-1]^2} \right) = 2 \frac{e^{[\mathfrak{x}-1]^2}}{\sqrt{\pi}} \frac{d}{dx} \mathfrak{x} \quad \text{Simplify}$$

$$\triangle \frac{d}{dx} \left(\frac{2}{\sqrt{\pi}} e^{[\mathfrak{x}-1]^2} \right) = 2 \left(\frac{\frac{1}{e^1} e^{\mathfrak{x}}}{\sqrt{\pi}} \left[\frac{d}{dx} \mathfrak{x} \right] \right) \quad \text{Expand}$$

$$\triangle \frac{d}{dx} \left(\frac{2}{\sqrt{\pi}} e^{[\mathfrak{x}-1]^2} \right) = 2 \frac{1}{\sqrt{\pi}} \frac{1}{e} e^{\mathfrak{x}} \frac{d}{dx} \mathfrak{x} \quad \text{Simplify}$$

$$\triangle \frac{d}{dx} \left(\frac{2}{\sqrt{\pi}} e^{[\mathfrak{x}-1]^2} \right) = 2 \frac{e^{\mathfrak{x}}}{e \sqrt{\pi}} \frac{d}{dx} \mathfrak{x} \quad \text{Simplify}$$

$$\square f''(1) = g'''(1)$$

$$\triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right] = \triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} g[x] \right) \right] \quad \text{Simplify}$$

$$\triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[2 \frac{\int_0^{x-1} e^{-t^2} dt}{\sqrt{\pi}} \right] \right) \right] = \triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} [a_1 x + a_0] \right) \right] \quad \text{Substi.}$$

$$\triangle_{x=1} \left[2 \frac{1}{\sqrt{\pi}} \frac{d}{dx} \frac{d}{dx} \left(\int_0^{x-1} e^{-t^2} dt \right) \right] = 0 \quad \text{Simplify}$$

$$\square 2 \frac{e^{\mathfrak{x}}}{e \sqrt{\pi}} = 0$$



$a_0 = -2 \frac{1}{\sqrt{\pi}}$

$a_1 = 2 \frac{1}{\sqrt{\pi}}$

$g(\mathfrak{x}) = a_1 \mathfrak{x} + a_0$

$$\triangle g(\mathfrak{x}) = \left(2 \frac{1}{\sqrt{\pi}} \right) \mathfrak{x} - 2 \frac{1}{\sqrt{\pi}} \quad \text{Substitute}$$



$\text{Erf}(\mathfrak{x}) = \frac{2}{\sqrt{\pi}} \left(\int_0^{\mathfrak{x}} e^{-t^2} dt \right)$

$f(\mathfrak{x}) = \text{Erf}(\mathfrak{x}-1)$

$$\triangle f(\mathfrak{x}) = \frac{2}{\sqrt{\pi}} \left(\int_0^{\mathfrak{x}-1} e^{-t^2} dt \right)$$

$$\triangle f(x) = 2 \frac{\int_0^{x-1} e^{-t^2} dt}{\sqrt{\pi}}$$

$f(x) = \frac{2}{\sqrt{\pi}} \left(\int_0^{x-1} e^{-t^2} dt \right)$

$g(x) = \left(2 \frac{1}{\sqrt{\pi}} \right) x - 2 \frac{1}{\sqrt{\pi}}$

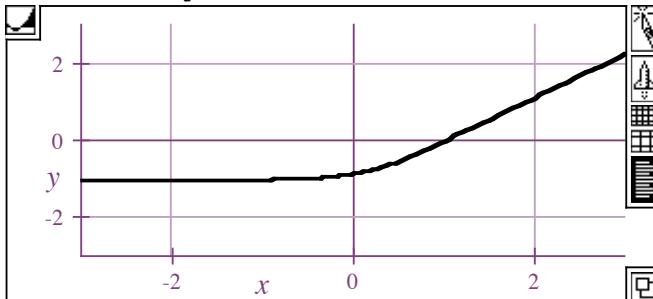
LS: 8/17/12: g(x) should have some numerical values in it, like $g(x) = 2x+3$ (although that's not it)
See hints above to get the a_1 and a_0 values.

LS: 8/24/12: You didn't have a "hot dot" on the $f(x)$ function, so it was pulling a $f(x)$ definition from elsewhere. I put the hotdot on your function, so it should look ok now.

8/27/12: Well... Thanks. Haha I hate it when the solution is a simple thing like that that you (or me) just miss.

?

$\text{spline}(x) = \begin{cases} f(x) & (x < 1) \\ g(x) & (x \geq 1) \end{cases}$

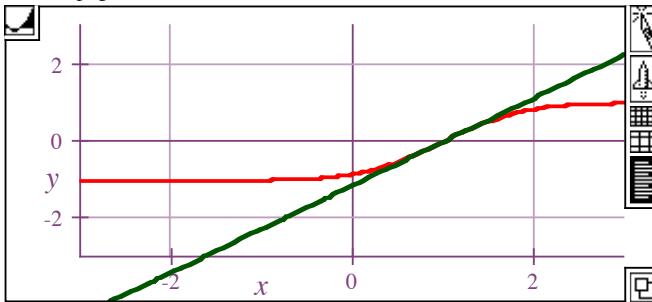


-3 ... 3 = left...right Stretch to Fit ▾

-3 ... 3 = bottom...top cropping Moderately ▾

Graph Building Blocks

Curve at $(x, \text{spline}[x])$ where $x = \text{left} \dots \text{right}$
with a heavy line, colored Black.



– 3 ... 3 = left...right Stretch to Fit ▼
 – 3 ... 3 = bottom...top cropping Moderately ▼

Graph Building Blocks

- ↳ Curve at $(x, f[x])$ where $x = \text{left} \dots \text{right}$ with a heavy line, colored Red.
- ↳ Curve at $(x, g[x])$ where $x = \text{left} \dots \text{right}$ with a heavy line, colored Dark Green.

8/14/12: Well.... that didn't work. Where did I go wrong here? Is it the a_0 term? Because I feel that I am wrong about that. I got it because when $f(1) = g(1)$, $a_0 + a_1 = 0$, so $a_1 = -a_0$, right?
 (Probably not...)

8/17/12: LS: 8/17/12: a_1 does equal $-a_0$, but you need to find the actual numbers for those.

8/23/12: That looked like it had more promise until I graphed it. It also seems that it would work if I added a bit more than 1 to $g(x)$.

G.2.d.ii)

8/27/12: Find the polynomial of degree 3 that has the highest possible order of contact with $f(x) = \text{Erf}(x - 1)$ at $x = 1$.

Plot the spline knotted at $\{1, 0\}$ with $f(x)$ on the right and your polynomial on the left. Also plot on the same axes $f(x)$, the polynomial, and the spline for $0 \leq x \leq 2$.

Describe what you see, paying special attention to a comparison of the plots in parts

8/27/12: LS: 8/17/12: This should go pretty much like 2.d.i, but you will have more terms in your polynomial.

8/27/12: LS: 8/24/12: Go ahead and try this one, now that d. i is correct.

8/27/12: Pre-problem comment - it should be quite "fun" attempting to find the right places to

pull everything from.

$$\boxed{\text{◻}} \quad \text{Erf}(\mathfrak{x}) = \frac{2}{\sqrt{\pi}} \left(\int_0^{\mathfrak{x}} e^{-t^2} dt \right)$$

$$\boxed{\text{◻}} \quad f(\mathfrak{x}) = \text{Erf}(\mathfrak{x} - 1)$$

$$\triangle f(\mathfrak{x}) = \frac{2}{\sqrt{\pi}} \left(\int_0^{\mathfrak{x}-1} e^{-t^2} dt \right)$$

$$\triangle f(\mathfrak{x}) = 2 \frac{\int_0^{\mathfrak{x}-1} e^{-t^2} dt}{\sqrt{\pi}}$$

$$\boxed{\text{◻}} \quad g(\mathfrak{x}) = \sum_{k=0}^3 a_k \mathfrak{x}^k$$

$$\triangle g(\mathfrak{x}) = a_0 \mathfrak{x}^0 + a_1 \mathfrak{x}^1 + a_2 \mathfrak{x}^2 + a_3 \mathfrak{x}^3 \quad \text{Expand}$$

$$\triangle g(\mathfrak{x}) = a_3 \mathfrak{x}^3 + a_2 \mathfrak{x}^2 + a_1 \mathfrak{x} + a_0 \quad \text{Simplify}$$

$$\boxed{\text{◻}} \quad f(1) = g(1)$$

$$\triangle 2 \frac{\int_0^{1-1} e^{-t^2} dt}{\sqrt{\pi}} = a_3 \cdot 1^3 + a_2 \cdot 1^2 + a_1 \cdot 1 + a_0 \quad \text{Substitute}$$

$$\triangle 2 \frac{\int_0^{1-1} e^{-t^2} dt}{\sqrt{\pi}} = a_3 + a_2 + a_1 + a_0 \quad \text{Simplify}$$

$$\boxed{\text{◻}} \quad 0 = a_3 + a_2 + a_1 + a_0$$

$$\triangle 0 = \frac{2}{3e\sqrt{\pi}} - 4 \frac{1}{e\sqrt{\pi}} + 8 \frac{1}{e\sqrt{\pi}} + a_0 \quad \text{Substitute}$$

$$\triangle 0 = a_0 + \frac{2}{3e\sqrt{\pi}} + 4 \frac{1}{e\sqrt{\pi}} \quad \text{Simplify}$$

$$\triangle a_0 = 0 - \left(\frac{2}{3e\sqrt{\pi}} + 4 \frac{1}{e\sqrt{\pi}} \right) \quad \text{Isolate}$$

$$\triangle a_0 = - \frac{2}{3e\sqrt{\pi}} - 4 \frac{1}{e\sqrt{\pi}} \quad \text{Simplify}$$

$$\boxed{\text{◻}} \quad f'(1) = g'(1)$$

$$\triangle \underset{x=1}{\left[\frac{d}{dx} f(x) \right]} = \underset{x=1}{\left[\frac{d}{dx} g(x) \right]} \quad \text{Simplify}$$

$$\triangle \underset{x=1}{\left[\frac{d}{dx} \left(2 \frac{\int_0^{x-1} e^{-t^2} dt}{\sqrt{\pi}} \right) \right]} = \underset{x=1}{\left[\frac{d}{dx} (a_3 x^3 + a_2 x^2 + a_1 x + a_0) \right]}$$

$$\triangle \underset{x=1}{\left[2 \frac{1}{\pi^{\frac{1}{2}}} \frac{d}{dx} \left(\int_0^{x-1} e^{-t^2} dt \right) \right]} = 3 a_3 \cdot 1^2 + 2 a_2 \cdot 1 + a_1 \quad \text{Simplify}$$

$$\triangle_{x=1} \left[2 \frac{1}{\sqrt{\pi}} \frac{d}{dx} \left(\int_0^{x-1} e^{-t^2} dt \right) \right] = 3a_3 + 2a_2 + a_1 \quad Simplify$$

$$\square \frac{2}{e\sqrt{\pi}} = 3a_3 + 2a_2 + a_1$$

$$\triangle \frac{2}{e\sqrt{\pi}} = 3a_3 + 2 \left(-4 \frac{1}{e\sqrt{\pi}} \right) + a_1 \quad Substitute$$

$$\triangle \frac{2}{e\sqrt{\pi}} = 3 \frac{2}{3e\sqrt{\pi}} + 2 \left(-4 \frac{1}{e\sqrt{\pi}} \right) + a_1 \quad Substitute$$

$$\triangle a_1 = \frac{2}{e\sqrt{\pi}} - \left(3 \frac{2}{3e\sqrt{\pi}} + 2 \left[-4 \frac{1}{e\sqrt{\pi}} \right] \right) \quad Isolate$$

$$\triangle a_1 = 8 \frac{1}{e\sqrt{\pi}} \quad Simplify$$

$$\square f''(1) = g''(1)$$

$$\triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} f[x] \right) \right] = \triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} g[x] \right) \right] \quad Simplify$$

$$\triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[2 \frac{\int_0^{x-1} e^{-t^2} dt}{\sqrt{\pi}} \right] \right) \right] = \triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} [a_3 x^3 + a_2 x^2 + a_1] \right) \right]$$

$$\triangle_{x=1} \left[2 \frac{1}{\sqrt{\pi}} \frac{d}{dx} \frac{d}{dx} \left(\int_0^{x-1} e^{-t^2} dt \right) \right] = 6a_3 + 2a_2 \quad Simplify$$

$$\triangle_{x=1} \left[2 \frac{1}{\sqrt{\pi}} \frac{d}{dx} \frac{d}{dx} \left(\int_0^{x-1} e^{-t^2} dt \right) \right] = 6a_3 + 2a_2 \quad Simplify$$

$$\square -\frac{4}{e\sqrt{\pi}} = 6a_3 + 2a_2$$

$$\triangle -\frac{4}{e\sqrt{\pi}} = 6 \frac{2}{3e\sqrt{\pi}} + 2a_2 \quad Substitute$$

$$\triangle a_2 = \frac{1}{2} \left(-\frac{4}{e\sqrt{\pi}} - 6 \frac{2}{3e\sqrt{\pi}} \right) \quad Isolate$$

$$\triangle a_2 = -4 \frac{1}{e\sqrt{\pi}} \quad Simplify$$

$$\square f'''(1) = g'''(1)$$

$$\triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} f\{x\} \right] \right) \right] = \triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} g\{x\} \right] \right) \right] \quad Simplify$$

$$\triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} \left(2 \frac{\int_0^{x-1} e^{-t^2} dt}{\sqrt{\pi}} \right) \right] \right) \right] = \triangle_{x=1} \left[\frac{d}{dx} \left(\frac{d}{dx} \left[\frac{d}{dx} [a_3 x^3 + a_2 x^2 + a_1] \right] \right) \right]$$

$$\triangle_{x=1} \left[2 \frac{1}{\sqrt{\pi}} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \left(\int_0^{x-1} e^{-t^2} dt \right) \right] = 6a_3 \quad Simplify$$

$$\square -\frac{4}{e\sqrt{\pi}} = 6a_3$$

$$\triangle a_3 = \frac{1}{6} \frac{4}{e\sqrt{\pi}} \quad Isolate$$

$$\triangle a_3 = \frac{2}{3e\sqrt{\pi}} \quad Simplify$$



$a_0 = -\frac{2}{3e\sqrt{\pi}} - 4\frac{1}{e\sqrt{\pi}}$

$a_1 = 8\frac{1}{e\sqrt{\pi}}$

$a_2 = -4\frac{1}{e\sqrt{\pi}}$

$a_3 = \frac{2}{3e\sqrt{\pi}}$

$g(x) = \sum_{k=0}^3 a_k x^k$

$$\triangle g(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 \quad Expand$$

$$\triangle g(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0 \quad Simplify$$

$$\triangle g(x) = \frac{2}{3e\sqrt{\pi}} x^3 + \left(-4\frac{1}{e\sqrt{\pi}}\right) x^2 + \left(8\frac{1}{e\sqrt{\pi}}\right) x + \left(-\frac{2}{3e\sqrt{\pi}} - 4\frac{1}{e\sqrt{\pi}}\right)$$

$$\triangle g(x) = -4\frac{x^2}{e\sqrt{\pi}} + 8\frac{x}{e\sqrt{\pi}} + \frac{2x^3}{3e\sqrt{\pi}} - \frac{2}{3e\sqrt{\pi}} - 4\frac{1}{e\sqrt{\pi}} \quad Simplify$$



$\text{Erf}(x) = \frac{2}{\sqrt{\pi}} \left(\int_0^x e^{-t^2} dt \right)$

$f(x) = \text{Erf}(x-1)$

$$\triangle f(x) = \frac{2}{\sqrt{\pi}} \left(\int_0^{x-1} e^{-t^2} dt \right)$$

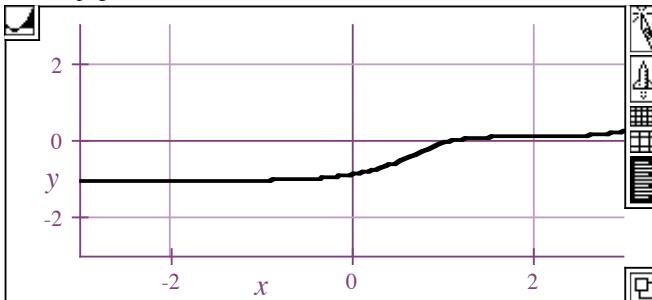
$$\triangle f(x) = 2 \frac{\int_0^{x-1} e^{-t^2} dt}{\sqrt{\pi}}$$

$$\square f(x) = \frac{2}{\sqrt{\pi}} \left(\int_0^x e^{-t^2} dt \right)$$

$g(x) = -4\frac{x^2}{e\sqrt{\pi}} + 8\frac{x}{e\sqrt{\pi}} + \frac{2x^3}{3e\sqrt{\pi}} - \frac{2}{3e\sqrt{\pi}} - 4\frac{1}{e\sqrt{\pi}}$

RC: 09/04/12: OK

$\text{spline}(x) = \begin{cases} f(x) & (x < 1) \\ g(x) & (x \geq 1) \end{cases}$



- 3 ... 3 = left...right Stretch to Fit▼
- 3 ... 3 = bottom...top cropping Moderately▼

Graph Building Blocks

Curve at $(x, \text{spline}[x])$ where $x = \text{left} \dots \text{right}$
with a heavy▼ line, colored Black▼.



- 3 ... 3 = left...right Stretch to Fit▼
- 3 ... 3 = bottom...top cropping Moderately▼

Graph Building Blocks

Curve at $(x, f[x])$ where $x = \text{left} \dots \text{right}$ with a
heavy▼ line, colored Red▼.

Curve at $(x, g[x])$ where $x = \text{left} \dots \text{right}$ with a
heavy▼ line, colored Dark Green▼.



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