Calculus III
(Series)
Detailed Syllabus

Mathematics

- Remarkable plots explained by order of contact. Splining for smoothness at the knots.
- The expansion of a function f[x] in powers of x as a file of polynomials with higher and higher orders of contact with f[x] at x = 0.
- The expansions every literate calculus person knows (1/(1 - x), ex, sin[x] and cos[x]).
- Expansions for approximations.
- The expansion of a function f[x] in powers of (x - b) as a file of polynomials with higher and higher orders of contact with f[x] at x = b.
- Newton's method.
- Multiplying and dividing expansions.
- Using expansions to help to calculate limits at a point.
- Expansions and the complex exponential function. Using expansions to help to get precise estimates of some integrals.

Science and Math Experience

- Experiments geared at discovering that the smoother the transition from one curve to another at a knot, the better both curves approximate each other near the knot.
- Splining functions and polynomials.
- Splines in road design.
- Landing an airplane.
- The natural cubic spline.
- Order of contact for derivatives and integrals.
- Experiments geared toward discovering that using more and more of the expansion results in better and better approximation.
- Halley's way of calculating accurate decimals.
- Expansions by substitution.
- Expansions by differentiation.
- Expansions by integration.
- Recognition of expansions.
- Expansions that satisfy a priori error bounds.
- Centering expansions for good approximation.
- Newton's method for root finding.
- Successes and failures of Newton's method.
- Using the complex exponential to generate trigonometric identities.
- Comparing reflecting properties of spherical mirrors and the relecting properties of parabolic mirrors.
- Using expansions to see why spherical mirror do have limited ability to concentrate light rays.
- Behavior of expansions very close to 0.
- Behavior of expansions far away from 0.

Mathematics

- Taylor's formula for expansions in powers of (x - b).
- Barriers and complex singularities.
- The convergence interval of an expansion as the interval between the barriers.
- Why some functions like $1/(1 + x^2)$ have barriers and others like e^x and sin[x] do not.
- Why functions like $x^{5/2}$ do not have expansions in powers of x but do have expansions in powers of (x - b) for b > 0.
- Why the barriers for f[x],f'[x] and f[t] dt are the same.

- Functions defined by a power series.
- Functions defined by power series via differential equations.
- The power series convergence principle, which says that if for some positive number r the infinite list $\{a_0, a_1 r, a_2 r^2, a_3 r^3, \ldots, a_k r^k, \ldots\}$ is bounded , then the power series $a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_k x^k + \ldots$ converges for $-r < x < r$.

Science and Math Experience

- Euler, Midpoint and Runge approximations of f[x] given f'[x]. Experiments comparing the quality of midpoint and Runge approximations.
- Adaption of Euler, midpoint and Runge approximations to approximating the plots of the differential equation y'[x] = f[x,y[x]] with y[a] = b. Taylor's formula in reverse.
- L'Hospital's rule by dividing the leading term of the expansion of the denominator into the leading term of the expansion of the numerator.
- Centering the expansion for best approximation.
- Experiments comparing the derivative of the expansion and the expansion of the derivative.
- Shortcuts based on the expansion of 1/(1 - x) in powers of x.
- Using the expansion of 1/(1 - x) in powers of x for drug dosing.
- Infinite sums of numbers resulting from expansions.
- Barriers resulting from splines.
- Infinite sums and decimals.
- Experiments relating expansions in powers of x to interpolating polynomials.
- Runge's disaster.
- Experiments in trying to plot functions defined by power series.
- Experiments in plotting a function defined by a power series via a differential equation versus plotting the same function directly through Mathematica's numerical differential equation solver. The ratio test for power series as a consequence of the power series convergence principle.
- The functions e^x, Sin[x] and Cos[x] from the viewpoint of power series.
- Experiments in truncation of power series.
- The Airy function as a function defined by a power series.

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